TOPIC 8 Gravitation

1 The SI unit for gravitational field is

A $m\ s^{-2}$
B $J\ kg^{-1}$
C $kg\ N^{-1}$
D $N\ m^{-1}$
E $N\ kg^{-2}\ m^2$ N76/II/6

2 A communications satellite which takes 24 hours to orbit the Earth is replaced by a new satellite which has twice the mass of the old one.

The new satellite also has an orbit time of 24 hours.

What is the value of $\frac{\text{radius of orbit of new satellite}}{\text{radius of orbit of old satellite}}$?

A $\frac{1}{2}$ B $\frac{1}{1}$ C $\frac{\sqrt{2}}{1}$ D $\frac{2}{1}$

J77/II/7; N94/I/8; J98/I/8

3 Assuming that the Earth is spherical and of radius r, its mean density is

A $\frac{4\pi rG}{3g}$
B $\frac{3rg}{4\pi G}$
C $\frac{4\pi rg}{3G}$
D $\frac{4\pi g}{3rG}$
E $\frac{3g}{4\pi rG}$

J77/II/8

4 The gravitational constant G has the SI unit

A $m\ s^{-2}$
B $N\ m^{-2}\ kg^{-2}$
C $m^3\ kg^{-1}\ s^{-2}$
D $m^2\ kg^{-2}$
E $J\ m\ kg^{-1}$ N77/II/2; J80/II/1

5 A satellite is in circular orbit 144 km above the Earth. Assuming the radius of the Earth to be 5760 km, the gravitational force on the satellite compared with that when it is at the Earth's surface is (approximately)

A greater by 10%
B greater by 5%
C the same
D less by 5%
E less by 10% N77/II/7

6 The values of the acceleration of free fall, g, on the surfaces of two planets will be the same provided that the planets have the same

A mass
B radius
C mass/radius
D mass/(radius)^2
E mass/(radius)^3 N77/II/8

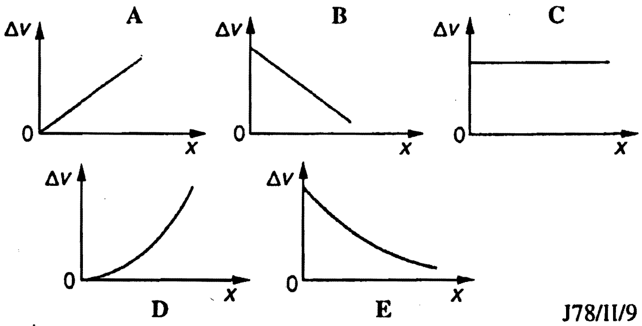
7 A planet has a mass of 5.0×10^{24} kg and a radius of 6.1×10^6 m. The energy needed to lift a mass of 2.0 kg from its surface into outer space is

A 9.0 J
B 1.8×10^1 J
C 5.5×10^7 J
D 1.1×10^8 J
E 2.2×10^8 J

[The gravitational constant, $G = 6.7 \times 10^{-11}\ N\ kg^{-2}\ m^2$.]

J78/II/8

8 Which one of the following graphs best indicates the relationship between ΔV, the change in gravitational potential energy of the hull of a spacecraft, and x, its height above the surface of the Earth, during the first few hundred metres after launch?



J78/II/9

9 A certain star of mass M and radius r rotates so rapidly that material at its equator only just remains on its surface. Given that the gravitational constant is G, the period of rotation is

A $2\pi\sqrt{(r/G)}$
B $2\pi\sqrt{(G/r)}$
C $2\pi\sqrt{(r/MG)}$
D $2\pi\sqrt{(r^3/MG)}$
E $2\pi\sqrt{(MG/r^3)}$

N78/II/8

10 A body of mass m is projected from the Earth's surface. At the point of launch, the acceleration of free fall is g and the radius of the Earth is R. To escape from the gravitational field of the Earth, the speed of the body must be at least

A $\sqrt{(gR)}$
B mgR
C $\sqrt{(2gR)}$
D $mg/2R$
E $\sqrt{(mgR)}$ J79/II/6

11 A body is moved from a point P on the Earth's surface to another point Q further from the Earth's centre. Which one of the following statements about the gravitational potential energy of the body at the two points is correct? [Take the gravitational potential energy of the body as zero when it is at an infinite distance from the Earth.]

A It is positive at both points and numerically greater at Q than at P.

B It is positive at both points and numerically less at Q than at P.

C It is zero at P but positive at Q.

D It is negative at both points and numerically greater at Q than at P.

E It is negative at both points and numerically less at Q than at P. J79/II/7

12 If a body of mass m were released in a vacuum just above the surface of a planet of mass M and radius R, what would be its gravitational acceleration?

A $\frac{GmM}{R}$ C $\frac{Gm}{R}$ E $\frac{GM}{R^2}$

B $\frac{GmM}{R^2}$ D $\frac{GM}{R}$ N79/II/1; J91/I/6

13 X and Y are two points at respective distances R and $2R$ from the centre of the Earth, where R is greater than the radius of the Earth. The gravitational potential at X is -800 kJ kg^{-1}. When a 1 kg mass is taken from X to Y, the work done on the mass is

A -400 kJ D $+400$ kJ

B -200 kJ E $+800$ kJ

C $+200$ kJ J80/II/5

14 An Earth satellite is moved from one stable circular orbit to another stable circular orbit at a greater distance from the Earth. Which one of the following quantities increases for the satellite as a result of the change?

A gravitational force

B gravitational potential energy

C angular velocity

D linear speed in the orbit

E centripetal acceleration N80/II/3; N86/I/6

15 A planet of mass P moves in a circular orbit of radius R round a sun of mass S with period T. Which one of the following correctly shows how T depends on P, R, S?

A $T \propto P^2$ D $T \propto S^{\frac{1}{2}}$

B $T \propto R^{\frac{1}{2}}$ E $T \propto S^2$

C $T \propto R^{\frac{3}{2}}$ N80/II/4

16 At a point on the surface of a uniform sphere of diameter d, the gravitational field due to the sphere is X. What would be the corresponding value on the surface of a uniform sphere of the same density but of diameter $2d$?

A $2X$ B $4X$ C $8X$ D $16X$ E $32X$ J81/II/8

17 An astronaut visits a planet of radius the same as that of the Earth. The acceleration of free fall at the surface of the planet is greater than that on Earth. Which one of the following will be the same as on Earth?

A the escape velocity of the astronaut

B the weight of the astronaut as measured by a spring balance

C the height to which the astronaut can jump

D the surface tension of a liquid

E the period of oscillation of a simple pendulum N81/II/3

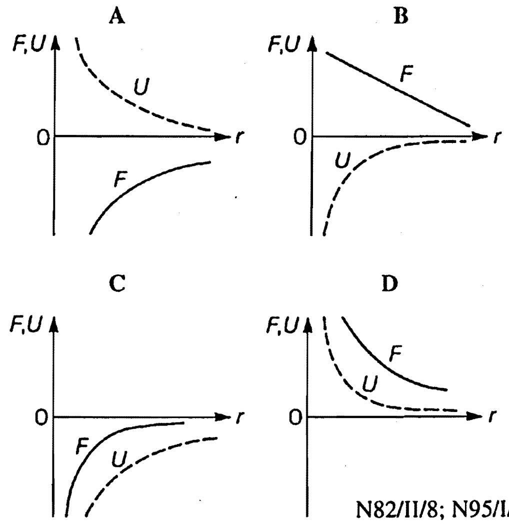
18 At a point outside the Earth and a distance x from its centre, the Earth's gravitational field is about 5 N kg^{-1}; at the Earth's surface, the field is about 10 N kg^{-1}. Which one of the following gives an approximate value for the radius of the Earth?

A $x/5$ D $x/\sqrt{2}$

B $x/2\sqrt{2}$ E $x\sqrt{2}$

C $x\sqrt{2}$ J82/II/8

19 Which diagram shows the variation of gravitational force F on a point mass, and of gravitational potential energy U of the mass, with its distance r from another point mass?



N82/II/8; N95/I/7

20 Assuming the Earth to be a uniform sphere rotating about an axis through the poles, the weight of a body at the Equator compared with its weight at a pole would be

A greater, because the angular velocity of the Earth is greater at the Equator than at a pole.

B greater, because the weight at the Equator is given by the sum of the gravitational attraction of the Earth and the centripetal force due to the circular motion of the body.

C the same, because the weight is the gravitational attraction of the Earth and for a uniform sphere, even when rotating, this is independent of the body's position on the Earth.

D smaller, because the gravitational attraction of the Earth must provide both the weight and the centripetal force due to the circular motion of the body.

E smaller, because the gravitational attraction at the pole is greater than that at the Equator. J83/II/6

21 The escape speed (i.e. the speed which a body must have in order to escape to an infinite distance from the Earth) of an oxygen molecule at the Earth's surface is 1.1×10^4 m s^{-1}. What is the escape speed at a height $0.2\ R_E$ above the Earth's surface, where R_E is the radius of the Earth?

A 0.5×10^4 m s^{-1}
B 1.0×10^4 m s^{-1}
C 1.1×10^4 m s^{-1}
D 1.2×10^4 m s^{-1}
E 1.3×10^4 m s^{-1} N83/II/8

22 According to one model X, the Earth is a solid sphere of uniform density. On another model Y, the Earth has a very dense core surrounded by less dense material.

The model are adjusted so that they give the same values of g, the acceleration of free fall at the Earth's surface. The values g_h at height h above the surface and g_d at depth d below the surface are also calculated on both models. Which one of the following correctly describes the results?

	g_h	g_d
A	same for both models	greater for X than Y
B	same for both models	smaller for X than Y
C	same for both models	same for both models
D	greater for X than Y	same for both models
E	smaller for X than Y	same for both models

J84/II/7

23 Two stars of equal mass M move with constant speed v in a circular orbit of radius R about their common centre of mass as shown in Fig. 1 below.

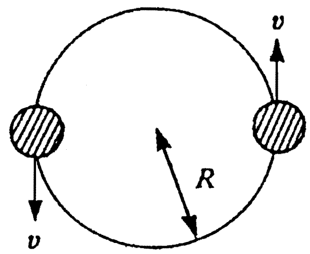


Fig. 1

What is the net force on each star?

A $GM^2/4R^2$
B $Mv^2/2R$
C zero
D $2Mv^2/R$
E GM^2/R^2 N84/II/7

24 On the ground the gravitational force on a satellite is W.

What is the gravitational force on the satellite when at a height $R/50$, where R is the radius of the Earth?

A 1.04 W
B 1.02 W
C 1.00 W
D 0.98 W
E 0.96 W J85/I/6; J90/I/8; N92/I/8

25 Star X of mass $2M$ and star Y of mass M perform circular motion about their common centre of mass under their gravitational attraction.

What is the ratio $\frac{\text{force acting on X}}{\text{force acting on Y}}$, ignoring the effects of any other bodies?

A 4 B 2 C 1 D $\frac{1}{2}$ E $\frac{1}{4}$

N85/I/5

26 In two widely-separated planetary systems whose suns have masses S_1 and S_2, planet P_1 of mass M_1 and planet P_2 of mass M_2 are observed to have circular orbits of equal radii. If P_1 completes an orbit in half the time taken by P_2 it may be deduced that

A $S_1 = S_2$ and $M_1 = 0.25\ M_2$.
B $S_1 = 4\ S_2$ only.
C $S_1 = 4\ S_2$ and $M_1 = M_2$.
D $S_1 = 0.25\ S_2$ only.
E $S_1 = 0.25\ S_2$ and $M_1 = M_2$. N85/I/6

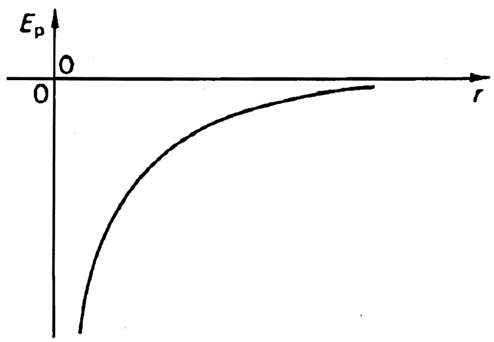
27 A satellite of mass m is in a circular orbit of radius r about the Earth, mass M, and remains at a vertical height h above the Earth's surface. Taking the zero of the gravitational potential to be at an infinite distance from the Earth, what is the gravitational potential energy of the satellite?

A mgh
B $-mgh$
C $-\frac{GMm}{r}$
D $\frac{-GMm}{2r}$
E $\frac{GMm}{2r}$ J86/I/18

28 *The Earth may be considered to be a uniform sphere of mass* M and radius R. Which one of the following equations correctly relates the universal gravitational constant G to the acceleration of free fall g at the surface of the Earth?

A $G = \frac{gM}{R^2}$
B $G = \frac{R^2}{gM}$
C $G = \frac{gR^2}{M}$
D $G = \frac{M}{gR^2}$
E $G = gMR^2$ J87/I/1

29 The gravitational potential energy E_p of a body varies with its distance r from the centre of a planet as shown in the diagram below.



What does the gradient at any point on the curve represent?

A the gravitational potential at that value of r

B the gravitational field strength at that value of r

C the force pulling the body towards the planet

D the acceleration of the body towards the planet

E the potential energy of the body at that value of r

J87/I/6

30 Which of the following is a property of a uniform gravitational field?

A Its magnitude is the same in all directions.

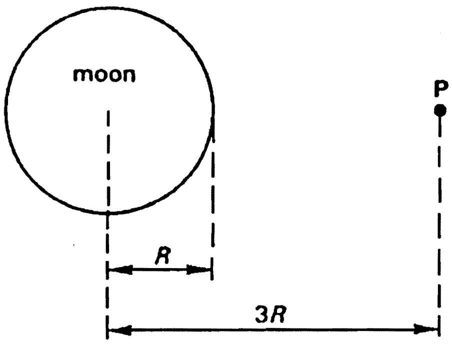
B The gravitational potential has the same value at all points within it.

C Its direction is opposite to the direction of motion of a test mass released in it.

D Its field strength is the same at all points within it.

J87/I/19; N94/I/7

31 A stationary object is released from a point **P** a distance 3 R from the centre of the Moon which has radius R and mass M.



Which one of the following expressions gives the speed of the object on hitting the Moon?

A $\left(\frac{2GM}{3R}\right)^{\frac{1}{2}}$

B $\left(\frac{4GM}{3R}\right)^{\frac{1}{2}}$

C $\left(\frac{2GM}{R}\right)^{\frac{1}{2}}$

D $\left(\frac{4GM}{R}\right)^{\frac{1}{2}}$

E $\left(\frac{GM}{R}\right)^{\frac{1}{2}}$

J88/I/18

32 Two stationary particles of masses M_1 and M_2 are a distance d apart. A third particle, lying on the line joining the particles, experiences no resultant gravitation force. What is the distance of this particle from M_1?

A $d\left(\frac{M_2}{M_1}\right)$

B $d\sqrt{\left(\frac{M_1}{M_2}\right)}$

C $d\sqrt{\left(\frac{M_1}{M_1+M_2}\right)}$

D $d\left(\frac{M_1}{M_1+M_2}\right)$

E $d\left(\frac{\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}\right)$

J88/I/19

33 A satellite of mass m is placed in an equatorial orbit so that it remains vertically above a fixed point on the Earth's surface.

If ω is the Earth's angular velocity of rotation and M is the Earth's mass, what is the radius of the satellite's orbit?

A $\left[\frac{GM}{\omega^2}\right]^{\frac{1}{3}}$

B $\left[\frac{Gm}{\omega^2}\right]^{\frac{1}{3}}$

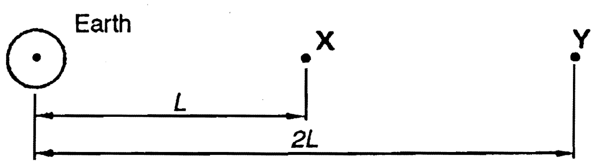
C $\left[\frac{GmM}{\omega^3}\right]^{\frac{1}{3}}$

D $\left[\frac{GM}{\omega^3}\right]^{\frac{1}{2}}$

E $\left[\frac{GmM}{\omega^2}\right]^{\frac{1}{2}}$

J89/I/7

34 The diagram shows two points **X** and **Y** at distances L and $2L$, respectively, from the centre of the Earth. The gravitational potential at **X** is -8 kJ kg^{-1}.



What is the gain in gravitational potential energy of a 1 kg mass when it is moved from **X** to **Y**?

A -4 kJ

B -2 kJ

C $+4$ kJ

D $+8$ kJ

N90/I/7; J97/I/7

35 A 20 kg mass is situated 4 m above the Earth's surface.

Taking g as 10 m s^{-2}, what are the gravitational field strength and gravitational force acting on the mass?

	gravitational field strength/ N kg^{-1}	*gravitational force*/N
A	0.5	10
B	10	10
C	10	200
D	40	200
E	200	10

N91/I/8

36 The acceleration of free fall on the surface of the Earth is 6 times its value on the surface of the Moon. The mean density of the Earth is $\frac{5}{3}$ times the mean density of the Moon.

If r_E is the radius of the Earth and r_M the radius of the Moon, what is the value of $\frac{r_E}{r_M}$?

A 1.9

B 3.6

C 6.0

D 10

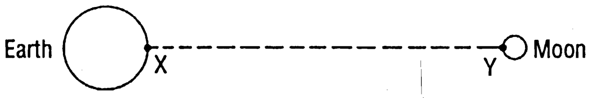
J92/I/8; J95/I/8

37 Why does the Moon stay in its orbit at a constant distance from the Earth?

A The gravitational pull of the Earth on the Moon is just sufficient to cause the centripetal acceleration of the Moon.
B The gravitational pull of the Earth on the Moon balances the gravitational pull of the Moon on the Earth.
C The gravitational pull of the Moon on the Earth is negligible at this distance.
D The centripetal force the Earth exerts on the Moon balances the centripetal force the Moon exerts on the Earth.
E The centripetal force the Earth exerts on the Moon balances the gravitational force the Earth exerts on the Moon.

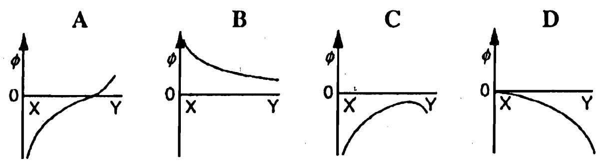
N92/I/7

38 The diagram (not to scale) represents the relative positions of the Earth and the Moon.



The line XY joins the surface of the Earth to the surface of the Moon.

Which graph represents the variation of gravitational potential ϕ along the line XY?



N93/I/5; N2000/I/7

39 A satellite of mass 50 kg moves from a point where the gravitational potential due to the Earth is –20 MJ kg^{-1}, to another point where the gravitational potential is –60 MJ kg^{-1}.

In which direction does the satellite move and what is its change in potential energy?

A closer to the Earth and a loss of 2000 MJ of potential energy.
B closer to the Earth and a loss of 40 MJ of potential energy.
C further from the Earth and a gain of 2000 MJ of potential energy.
D further from the Earth and a gain of 40 MJ of potential energy.

J94/I/7; N99/I/7

40 For points outside a uniform sphere of mass M, the gravitational field is the same as that of a point mass M at the centre of the sphere. The Earth may be taken to be a uniform sphere of radius r and density ρ.

How is the gravitational field strength g at its surface related to these quantities and the gravitational constant G?

A $g = \dfrac{G\rho}{r^2}$ **C** $g = \dfrac{4\pi r\rho G}{3}$

B $g = \dfrac{3G}{4\pi r\rho}$ **D** $g = \dfrac{4\pi r^2\rho G}{3}$

J96/I/7

41 The gravitational field strength at a point **P** on the Earth's surface is numerically equal to

A the acceleration of free fall at **P**.
B the change in potential energy per unit distance from **P**.
C the force acting on any body placed at **P**.
D the work done in bringing unit mass from infinity to **P**.

N96/I/8

42 An experimental satellite is found to have a weight W when assembled before launching from a rocket site. It is placed in a circular orbit at a height $h = 6R$ above the surface of the Earth (of radius R).

What is the gravitational force acting on the satellite whilst in orbit?

A $\dfrac{W}{6}$ **B** $\dfrac{W}{7}$ **C** $\dfrac{W}{36}$ **D** $\dfrac{W}{49}$

N97/I/7

43 Two point masses m_1 and m_2 are a distance r apart.

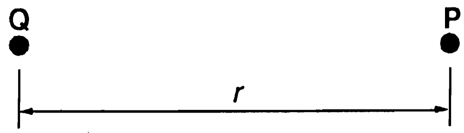
What is the magnitude of the gravitational field strength caused by m_1 at m_2?

A $\dfrac{Gm_1m_2}{r}$ **C** $\dfrac{Gm_1}{r^2}$

B $\dfrac{Gm_1m_2}{r^2}$ **D** $\dfrac{Gm_2}{r^2}$

J98/I/7

44 A mass m is at fixed point **Q**. It produces a gravitational potential at point **P**, distant r from **Q**.



This gravitational potential is equal to the external work done on unit mass in moving it

A from **P** to **Q**. **C** from **P** to infinity.
B from **Q** to **P**. **D** from infinity to **P**.

N98/I/7

45 The Earth experiences gravitational forces from the Sun, mass M_S, and from the Moon, mass M_m. The distance of the Sun from the Earth is r_S and the distance of the Moon from the Earth is r_m.

What is the ratio $\dfrac{\text{force on the Earth due to the Sun}}{\text{force on the Earth due to the Moon}}$?

A $\dfrac{M_S}{M_m}\left(\dfrac{r_S}{r_m}\right)$ **C** $\dfrac{M_S}{M_m}\left(\dfrac{r_S}{r_m}\right)^2$

B $\dfrac{M_S}{M_m}\left(\dfrac{r_m}{r_S}\right)$ **D** $\dfrac{M_S}{M_m}\left(\dfrac{r_m}{r_S}\right)^2$

J99/I/7

46 Outside a uniform sphere of mass M, the gravitational field strength is the same as that of a point mass M at the centre of the sphere.

The Earth may be taken to be a uniform sphere of radius r. The gravitational field strength at its surface is g.

What is the gravitational field strength at a height h above the ground?

A $\dfrac{gr^2}{(r+h)^2}$ **C** $\dfrac{g(r-h)}{r}$

B $\dfrac{gr}{(r+h)}$ **D** $\dfrac{g(r-h)^2}{r^2}$ J2000/I/7

47 Which quantity is **not necessarily** the same for satellites that are in geostationary orbits around the Earth?

A angular velocity **C** kinetic energy
B centripetal acceleration **D** orbital period

J2000/I/8

48 Which statement about geostationary orbits is **false**?

A A geostationary orbit must be directly above the equator.
B All satellites in a geostationary orbit must have the same mass.
C The period of a geostationary orbit must be 24 hours.
D There is only one possible radius for a geostationary orbit. N2000/I/8

49 A space capsule is travelling between the Earth and the moon. Find the distance from the Earth at which it is subject to zero gravitational force. (Consider only the gravitational fields of the Earth and the Moon.)

[Mass of the Earth = 6.0×10^{24} kg; mass of the Moon = 7.4×10^{22} kg; distance between the centres of the Earth and Moon = 3.8×10^{8} m.] J76/I/2

50 Assuming the Earth to be a sphere of radius 6×10^6 m, estimate the mass of the Earth, given that the acceleration of free fall is 10 m s^{-2} and that the gravitational constant G is 7×10^{-11} N m^2 kg^{-2}. N77/I/2

51 The speed with which a body should be projected from the Earth's surface in order to reach an infinite distance is about 1.1×10^4 m s^{-1}. Estimate the speed of escape from the moon.

[(Mass of Earth)/(mass of Moon) = 81;
(radius of Earth/(radius of Moon) = 3.7.] N79/I/1

52 The mass of the Earth is about 80 times that of the Moon, and the radius of the Earth is about 3.7 times that of the Moon. Taking the value of the acceleration of free fall on Earth to be 10 m s^{-2}, estimate its value on the Moon. N80/I/3

53 The radius R_E of the Earth is 6.4×10^6 m and the acceleration of free fall at its surface is 9.8 m s^{-2}. Find the value of the acceleration of free fall at an attitude of 6.4×10^5 m (i.e. at a distance of 1.1 R_E from the centre of the Earth.)

N82/I/2

54 What are the gravitational potentials at a point on the Earth's surface due to *(a)* the Earth, *(b)* the Sun?

[Mass of Earth = 6.0×10^{24} kg; radius of Earth = 6.4×10^6 m; mass of Sun = 2.0×10^{30} kg; radius of Earth's orbit = 1.5×10^{11} m.]
J83/I/1

55 Astronomical observations show that the centre of mass of the Earth-Moon system is 4.7×10^6 m from the centre of the Earth. The distance between the centres of the Earth and the Moon is 384.4×10^6 m. Find the mass of the Moon M_M in terms of the mass of the Earth M_E.

Explain why both Earth and Moon must rotate about their common centre of mass, rather than the moon about the centre of mass of the Earth. J84/I/2

56 A mass of 2 kg is at a point **P**, a height 3 m above the surface the Earth. Taking the gravitational potential at the surface of the Earth to be zero, state

(a) the gravitational field strength at **P**,
(b) the gravitational potential at **P**,
(c) the gravitational force acting on the mass,
(d) the gravitational potential energy of the mass. [6]

J88/II/2

57 Find the speed of a satellite which orbits the Moon near the Moon's surface. What is the kinetic energy per unit mass of the satellite?

[Radius of the moon = 1.74×10^6 m;
mass of the moon = 7.35×10^{22} kg.] N88/II/3

58 Values for the gravitational potential due to the Earth are given in the table below.

Distance from Earth's surface /m	*Gravitational potential* /MJ kg^{-1}
0	–62.72
390 000	–59.12
400 000	–59.03
410 000	–58.94
Infinity	0

(i) If a satellite of mass 700 kg falls from a height of 400 000 m to the Earth's surface, how much potential energy does it lose?

(ii) Deduce a value for the Earth's gravitational field at a height of 400 000 m. [5] J89/II/2

59 Fig. 2 shows a planet **P** of mass m orbiting the Sun **S** of mass M in a circular path of radius r.

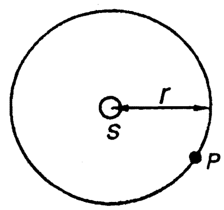


Fig. 2

(a) Write down an expression, in terms of G, m, M and r, for the force exerted by the Sun on the planet. [1]

(b) Use this expression to find the angular velocity of the planet in its orbit. [2]

(c) Deduce the time taken to complete one orbit of the Sun. [2]

(d) The Earth is 1.50×10^{11} m from the centre of the Sun and takes exactly one year to complete one orbit. The planet Jupiter takes 11.9 years to complete an orbit of the Sun. Calculate the radius of Jupiter's orbit. [4]

J90/II/2

60 (a) Given an expression for *Newton's law of Gravitation*, explaining the symbols you use. [2]

(b) Show that g, the gravitational field strength a height h above the surface of a uniform planet of mass M and radius R, is given by

$$g = \frac{GM}{(R+h)^2}$$ [2]

(c) Information related to the Earth and the Moon is given below.

$$\frac{\text{Radius of Earth}}{\text{Radius of Moon}} = 3.7$$

$$\frac{\text{Mass of Earth}}{\text{Mass of Moon}} = 81$$

Distance of Moon from Earth = 3.84×10^8 m.

Gravitational field strength due to the Earth at its surface = $9.8\ \text{N kg}^{-1}$.

(i) Using these data, calculate the gravitational field strength due to the Moon at its surface. [2]

(ii) There is a point on the line between the Earth and the Moon at which their combined gravitational field strength is zero. Calculate the distance between this point and the centre of the Earth. [3]

N90/II/2

61 (a) Two small masses m_1 and m_2 are placed at **X** and **Y** respectively and are separated by a distance r as shown in Fig. 3.

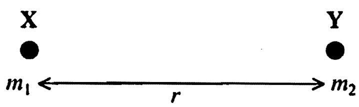


Fig. 3

(i) Draw on Fig. 3 the direction of the gravitational field which m_1 causes at **Y**.

(ii) What is the value of the gravitational field strength which m_1 causes at **Y**?

(iii) What is the force which m_1 causes on m_2? [3]

J92/II/1 (part)

62 A planet P of mass m orbits the Sun S of mass M in a circular orbit of radius r with angular velocity ω as shown in **Fig. 4**.

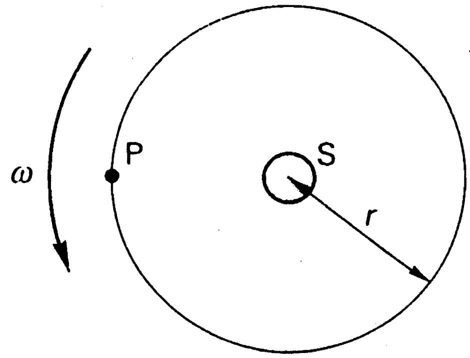


Fig. 4

(a) On the diagram of **Fig. 4**, draw an arrow representing the linear velocity of P and label this v.

Draw a second arrow representing the direction of the force acting on P. Label this F. [2]

(b) (i) Write down an expression, in terms of r and ω, for the magnitude of v.

(ii) Write down an expression, in terms of m, r and ω, for the magnitude of F.

(iii) Write down an expression, in terms of m, M, r and G, for the magnitude of the gravitational force exerted by the Sun on the planet.

(c) From observations of the motion of the planets around the Sun, Kepler (1571 – 1630) found that T^2, the square of the period of revolution of a planet around the Sun, was proportional to r^3.

(i) Write down an expression for T in terms of the angular velocity ω of the planet.

(ii) Use your answers to (b) (ii), (b) (iii) and (c) (i) to show that Kepler's relation,

$$T^2 \propto r^3,$$

would be expected. [3]

J93/II/1

63 (a) Define the term *gravitational field strength*. [1]

(b) State the numerical value and the unit of the gravitational field strength of the Earth at its surface. [2]

(c) Why is it incorrect to call g (= $9.8\ \text{m s}^{-2}$) 'gravity'? [2]

(d) This part of the question is about the rotation of the Moon in a circular orbit around the Earth. You will need to use the following astronomical data.

Radius of the Moon's orbit = 3.84×10^8 m
Mass of the Moon = 7.35×10^{22} kg
Time for Moon to complete one orbit around the Earth = 2.36×10^6 s

Calculate

(i) the speed of the Moon in its orbit around the Earth,

(ii) the acceleration of the Moon,

(iii) the force the Earth exerts on the Moon,

(iv) the gravitational field strength of the Earth at the Moon. [6]

J94/II/2

64 The mass of the Earth is 5.98×10^{24} kg and its mean radius is 6.37×10^6 m.

(a) Use Newton's law of gravitation to calculate the gravitational force acting on a 1.00 kg mass on the surface of the Earth. Assume that the Earth acts as a point mass.

gravitational force = ... N [3]

(b) State the value of the Earth's gravitational field strength at its surface.

field strength = ... [1]

(c) What is meant by the *gravitational potential* at a point in a gravitational field? [2]

(d) Calculate the difference in the gravitational potential between the surface of the Earth and a point 800 m above the surface.

difference in potential = ... [3]

J98/II/2

Long Questions

65 Explain what is meant by *gravitational field strength,* and show that it has the same dimensions as acceleration.

Sketch two graphs on the same horizontal scale of r to show how *(a)* the gravitational field strength, *(b)* the gravitational potential, vary with distance r from a point mass. Explain how the curves are related.

Find the speed with which a rocket must be projected from the Earth's surface in order to reach an infinite distance from the Earth.

What explanation can you offer for the fact that practical space vehicles are launched by multi-stage rockets, fired an intervals along the trajectory, rather than by a single rocket ignited at the Earth 's surface?

[$g = 10$ m s^{-2}; radius of the Earth = 6.4×10^6 m.] J77/I/14

66 Explain what is meant by (i) *gravitational field strength,* (ii) *gravitational potential.* How do these quantities vary with distance x from a point mass?

It can be shown that, for planets performing circular orbits about a sun, the period of revolution τ is related to the radius r of the orbit by the equation

$$\tau = A r^n$$

where A and n are constants. Values of τ and the mean radius r for some of the planets of our solar system are given in the table below.

Planet	τ/year	$r/10^6$ km
Mercury	0.241	58
Venus	0.615	108
Mars	1.88	228
Jupiter	11.9	778
Saturn	29.5	1430

(a) By drawing a suitable graph, deduce the values of A and n.

(b) Find the mean radius of the Earth's orbit. N78/I/14

67 The variation of the acceleration of free fall g over the Earth's surface may be measured by finding the period of a simple pendulum at various places. Explain why, even if the Earth were assumed to be a perfect sphere of uniform density, different periods would be obtained at a pole and at the equator. Estimate the percentage difference in the periods from the following approximate data:

acceleration of free fall, $g = 10$ m s^{-2},
radius of Earth = 6.4×10^6 m,
1 day = 8.6×10^4 s,
$\pi^2 = 10$.

Discuss whether you could detect this difference using a stopwatch or stopclock available in your school laboratory.

N79/I/4 (part)

68 State *Newton's law of gravitation.*

Derive an expression for the mass M of the Earth in terms of the gravitational constant G, the acceleration of free fall g and the Earth's radius r_E. What assumptions have you made about the Earth in this derivation?

A communications satellite of mass 50 kg is to be put into an equatorial orbit in which it has an angular velocity equal to that at which the Earth rotates about its axis, so that the satellite remains above the same point on the Earth's surface.

(a) What is the angular velocity of the Earth's rotation about its axis? (Give your answer in rad s^{-1}.)

(b) Find the radius of the satellite's orbit.

(c) By how much would (i) the potential energy, (ii) the kinetic energy, of the satellite change if it were moved from a position at rest on the Earth's surface to the required orbit?

[1 day = 8.4×10^4 s; radius of Earth = 6.4×10^6 m; mass of Earth = 6.0×10^{24} kg.]

N81/I/14

69 *(b)* Assuming that the planets move in circular orbits about the Sun, show that the squares of their periods of revolution are proportional to the cubes of the radii of their orbits. Upon what factors does the constant of proportionality depend?

The radius of the Earth's orbit is 1.5×10^{11} m and the Earth's period of revolution about the Sun is 3.2×10^7 s. The distance of the Moon from the Earth is 3.8×10^6 m and it makes one revolution about the Earth in 2.4×10^6 s. Find the ratio of the mass of the Sun to that of the Earth.

J82/I/15 (part)

70 Write down an expression for the gravitational potential energy of a body of mass m at a distance r from the centre of the Earth, of mass M_E. (Consider only values of r greater than R_E, the radius of the Earth.)

Show that body projected from the Earth (assumed to be stationary) with a speed equal to or greater than the escape speed $\sqrt{2g R_E}$ will never return. Give two assumptions necessary for this result to be valid.

The table below gives approximate values of the radius R and mass M of the Earth, Sun and Moon.

	R/m	M/kg
Earth	6×10^6	6.0×10^{24}
Sun	7.0×10^8	2.0×10^{30}
Moon	1.7×10^6	7.4×10^{22}

(a) Given that the escape speed from Earth is 1.1×10^4 m s^{-1}, estimate the escape speeds from the Sun and the Moon.

(b) One theory of atmospheric evolution suggests that the Earth originally had an atmosphere rich in hydrogen but that, as a result of a major thermal event in which the temperature rose to about 6 000 K, the hydrogen concentration then fell to its present very low level. Making reference to the distribution of molecular speeds, explain how this increase of temperature could have led to a substantial loss of hydrogen. (The r.m.s. speed of hydrogen atoms at 6 000 K is about 1.2×10^4 m s^{-1}.)

The surface temperature of the Sun is also about 6 000 K but hydrogen is the most abundant element in the Sun's atmosphere. Why do hydrogen atoms escape much less readily from the Sun than from the Earth?

On the Moon, the concentrations of all gases are so low that it has effectively no atmosphere. Suggest an explanation.

N84/I/15

71 State the law describing the gravitational force between two point masses M and m a distance r apart.

Two alternative units for gravitational field strength are N kg^{-1} and m s^{-2}. Use the method of dimensions to show that they are equivalent.

State the general relationship between the field strength at a point in a field of force and the potential gradient at that point. Write down an expression for the gravitational potential at a point distance r from a mass M. Distinguish between *gravitational potential* and *gravitational potential energy*.

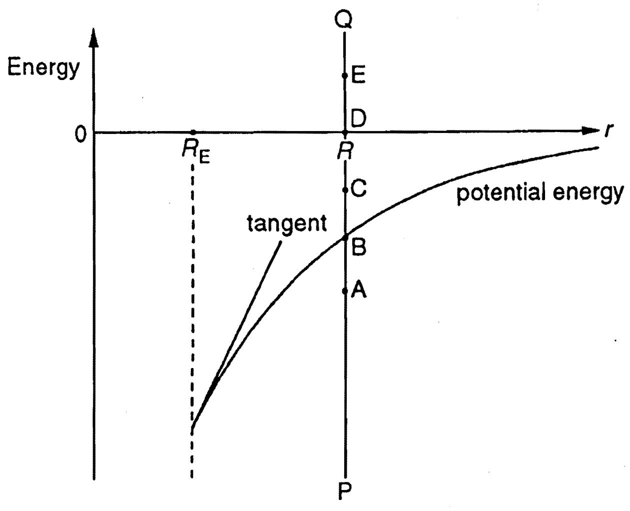


Fig. 5

The curve in Fig. 5 shows the way in which the gravitational potential energy of a body of mass m in the field of the Earth depends on r, the distance from the centre of the Earth, for values of r greater than the Earth's radius R_E. What does the gradient of the tangent to the curve at $r = R_E$ represent?

The body referred to above is a rocket which is projected vertically upwards from the Earth. At a certain distance R from the centre of the Earth, the total energy of the rocket (i.e. its gravitational potential energy plus its kinetic energy) may be represented by a point on the line PQ. Five points A, B, C, D, E have been marked on this line. Which point (or points) could represent the total energy of the rocket

(a) if it were momentarily at rest at the top of its trajectory,

(b) if it were falling towards the Earth,

(c) if it were moving away from the Earth, with sufficient energy to reach an infinite distance?

In each case, explain briefly how you arrive at your answer.

J85/II/9

72 A point mass m is at a distance r from the centre of the Earth. Write down an expression, in terms of m, r, the Earth's mass m_E and the gravitational constant G, for the gravitational potential energy V of the mass. (Consider only values of r greater than the Earth's radius).

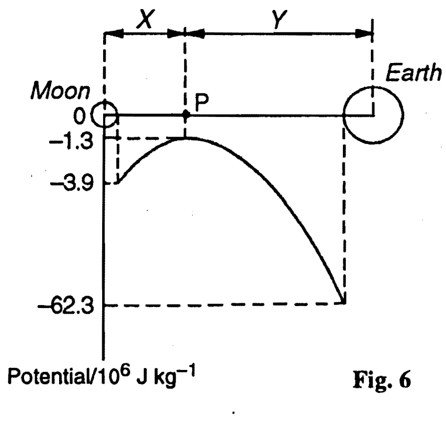


Fig. 6

Certain meteorites (tektites) found on Earth have a composition identical with that of lunar granite. It is thought that they may be debris from a volcanic eruption on the Moon. Fig. 6 which is not to scale, shows how the gravitational potential between the surface of the Moon and the surface of the Earth varies along the line of centres. At the point P the gravitational potential is a maximum.

(a) By considering the separate contributions of Earth and Moon to the gravitational potential, explain qualitatively why the graph has a maximum and why the curve is asymmetrical.

(b) State how the resultant gravitational force on the tektite at any point between the Moon and the Earth could be deduced from Fig. 6.

(c) When a tektite is at P the gravitational forces on it due to Moon and Earth are F_M and F_E respectively. State the relation which applies between F_M and F_E. Hence find the values of x/y, where x and y are the distances of P from the centre of the Moon and the centre of the Earth respectively.

(d) If a tektite is to reach the Earth, it must be projected from the volcano on the Moon with a certain minimum speed V_O. Making use of appropriate values from Fig. 6, find this speed. Explain your reasoning.

(e) Discuss very briefly whether a tektite will reach the Earth's surface with a speed less than, equal to or greater than the speed of projection. (Neglect atmospheric resistance.)

(Mass of Moon = 7.4×10^{22} kg;
mass of Earth = 6.0×10^{24} kg.) J87/II/8

73 Define *gravitational field strength* and *gravitational potential* at a point in a gravitational field, and state units in which each of these quantities may be measured. [4]

The gravitational potential energy V of a body of mass m in the field of the Earth is given by $V = -GM_E m/r$, where G is the gravitational constant, M_E is the mass of the Earth and r is the distance of the body from the centre of the Earth. If this expression is to hold, what assumption must be made about the Earth? For what values of r is the expression valid? Explain why the potential energy, in this case, is negative. [4]

A satellite of mass m moves in a circular orbit about the Earth.

(a) Derive an expression for its kinetic energy T in terms of G, M_E, m and the radius r of the orbit. Hence show that $T = -\frac{1}{2}V$, where V is the potential energy of the satellite. Write down the relative between T and the total energy E of the satellite. [6]

(b) Over a period of time atmospheric friction reduces the total energy of the satellite by ΔE. The change is so gradual that the orbit may be assumed to remain circular. Find in terms of ΔE, the corresponding changes in the satellite's kinetic energy T and potential energy V. (State clearly whether each change is an increase or a decrease.) Describe in words the effect of atmospheric resistance on the motion of the satellite. [3]

N87/II/9

74 *(b)* What is the acceleration of the Moon? The Moon may be considered to travel about the Earth in a circular orbit of radius 3.82×10^8 m and period 2.36×10^6 s. Why does the Moon not fall and hit the Earth? [5]

By considering the acceleration of free fall at the Earth's surface, show that the magnitude of the Moon's acceleration is consistent with Newton's inverse square law of gravitation. [6]
[Radius of the Earth = 6.36×10^6 m.] J88/II/9 (part)

75 *(b)* Modern gravity meters can measure g, the acceleration of free fall, to a high degree of accuracy. The principle on which they work is of measuring t, the time of fall of an object through a known distance h in a vacuum. Assuming that the object starts from rest, deduce the relation between g, t and h. [2]

(c) State Newton's law of gravitation relating the force F between two point objects of masses m and M, their separation r and the gravitational constant G. [2]

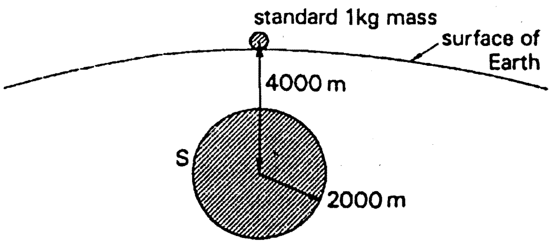


Fig. 7 Not to scale

(d) Fig. 7 shows a standard kilogram mass at the surface of the Earth and a spherical region S of radius 2000 m with its centre 4000 m from the surface of the Earth. The density of the rock in this region is 2800 kg m^{-3}. What force does the matter in region S exert on the standard mass? [3]

(e) If region S consisted of oil of density 900 kg m^{-3} instead of rock, what difference would there be in the force on the standard mass? [3]

(f) Suggest how gravity meters may be used in oil prospecting. Find the uncertainty within which the acceleration of free fall needs to be measured if the meters are to detect the (rather large) quantity of oil stated in *(e)*. [4]

J91/III/1 (part)

76 *(a)* (i) State *Newton's law of gravitation.*

(ii) The first definition of the metre was one ten-millionth of the distance between the north pole and the equator of the Earth. Use this information to estimate the radius of the Earth. State one assumption which you have made in your estimation.

(iii) Use your answers to (i) and (ii) to deduce the gravitational force acting on a 1.0 kg mass at the Earth's surface. The Earth may be considered to be a sphere of mass 6.0×10^{24} kg. [6]

(b) (i) What gravitational force does the Earth exert on a 1.0 kg mass which is at a distance of 3.8×10^{8} m from the centre of the Earth? [2]

(ii) Assuming that the Moon travels at a constant speed around the Earth in a circle of this radius (3.8×10^{8} m), with the Earth at the centre of the circle, calculate the acceleration of the Moon. Show the direction of this acceleration on a sketch diagram. [4]

(iii) Calculate the period of the Moon's rotation. [3]

(c) A satellite orbits the Earth in a circular path of radius *r*. The period of the orbit is *T*.

(i) Find the acceleration of the satellite.

(ii) Hence show that

$$T^2 = \frac{4\pi^2}{GM} r^3 ,$$

where *M* is the mass of the Earth. [5] N93/III/2

77 *(a)* The gravitational field strength of the Earth at its surface is 9.81 N kg^{-1}.

Show that

(i) the acceleration of free fall at the surface of the Earth is 9.81 m s^{-2},

(ii) N kg^{-1} is equivalent to m s^{-2} in base units. [4]

(b) Use the value for the gravitational field strength of the Earth quoted in *(a)*, together with the value of *G*, the gravitational constant, and of the radius of the Earth (6.38×10^{6} m), to calculate the mass of the Earth. [4]

(c) Calculate the Earth's gravitational field strength at a height of 0.12×10^{6} m above the Earth's surface. [3]

(d) Explain briefly why an astronaut in a satellite orbiting the Earth at this altitude may be described as weightless. [2]

(e) The value of the gravitational potential ϕ at a point in the Earth's field is given by the equation

$$\phi = -GM/r,$$

where *M* is the mass of the Earth and *r* is the distance of the point from the centre of the Earth. (*r* is greater than the radius of the Earth.)

Explain

(i) what is meant by the term *gravitational potential*,

(ii) why the potential has a negative value. [3]

(f) Use the expression given in *(e)* to calculate the gain in the potential energy of a satellite of mass 3000 kg between its launch and when it is at a height of 0.12×10^{6} m above the Earth's surface. [4]
N94/III/1

78 *(c)* The Earth may be considered to be a uniform sphere of radius 6370 km, spinning on its axis with a period of 24.0 hours. The gravitational field at the Earth's surface is identical with that of a point mass of 5.98×10^{24} kg at the Earth's centre. For a 1.00 kg mass situated at the Equator,

(i) calculate, using Newton's law of Gravitation, the gravitational force on the mass,

(ii) determine the force required to maintain the circular path of the mass,

(iii) deduce the reading on an accurate newton-meter (spring balance) supporting the mass. [6]

(d) Using your answers to *(c)*, state what would be the acceleration of the mass at the Earth's surface due to

(i) the gravitational force alone,

(ii) the force as measured on the newton-meter. [2]

(e) A student, situated at the Equator, releases a ball from rest in a vacuum and measures its acceleration towards the Earth's surface. He then states that this acceleration is 'the acceleration due to gravity'. Comment on his statement. [2]
J96/III/2 (part)

79 The following astronomical data are required in answering this question.

Mass of the Earth	5.98×10^{24} kg
Mass of the Moon	7.35×10^{22} kg
Radius of the Earth (assumed constant)	6.378×10^{6} m
Distance from the centre of the Earth to the centre of the Moon (assumed constant)	3.84×10^{8} m.

(a) The metre was originally defined so that the distance along the Earth's surface from the equator to the North pole was 10 000 km. What percentage error is there between this original definition of the metre and the modern definition, which was used when quoting the radius of the Earth? [3]

(b) Use Newton's law of gravitation to calculate the gravitational force which the Earth exerts on the Moon. [3]

(c) Calculate the acceleration of the Moon due to the force in *(b)*. State the direction of this acceleration and explain why this acceleration does not increase the speed of the Moon. [4]

(d) Use your answer to *(c)* to deduce the angular velocity of the Moon about the Earth and the period of it in its orbit. [4]

(e) Show that the period *T* of a satellite orbiting the Earth in a circle of radius *r* is given by

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

where G is the gravitational constant and M is the mass of the Earth [4]

(*f*) Use the equation in *(e)* to find the radius of the orbit of a geostationary satellite. [2]

J97/III/2

80 *(a)* (i) Define *gravitational field strength*.

(ii) State a unit for gravitational field strength.

(iii) The gravitational field strength near the surface of the Earth is also known as the acceleration of free fall. Use base units to check that the unit of gravitational field strength is the same as that of acceleration [4]

(b) (i) State an equation to represent Newton's law of gravitation, and explain the symbols used.

(ii) Use Newton's law of gravitation and the definition of gravitational field strength to derive an expression for the gravitational field strength g at a distance r from a point mass M.

(iii) At any point above the surface of the Earth, the Earth may be assumed to be a point mass situated at its centre. Explain why the acceleration of free fall is approximately constant between the Earth's surface and a point about 1000 m above it. [5]

J99/III/3 (part)

81 *(a)* Explain how an object travelling in a circle with constant speed has an acceleration. In what direction is this acceleration? [4]

(b) A satellite P of mass 2400 kg is placed in a geostationary orbit at a distance of 4.23×10^7 m from the centre of the Earth.

(i) Explain what is meant by the term *geostationary orbit*. [1]

(ii) Calculate

1. the angular velocity of the satellite,

2. the speed of the satellite,

3. the acceleration of the satellite,

4. the force of attraction between the Earth and the satellite,

5. the mass of the Earth. [10]

(c) Explain why a geostationary satellite

(i) must be placed vertically above the equator,

(ii) must move from west to east. [4]

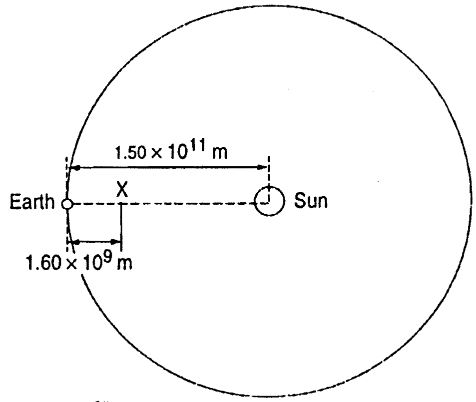
(d) Why is a satellite in a geostationary orbit often used for telecommunications? [1]

N99/III/2

82 *(a)* (i) Define *angular velocity* for an object travelling in a circle.

(ii) Calculate the angular velocity of the Earth in its orbit around the Sun. Assume that the orbit is circular and give your answer in terms of the SI unit for angular velocity. [4]

(b) In order to observe the Sun continuously, a satellite of mass 425 kg is at point X, a distance of 1.60×10^9 m from the centre of the Earth, as shown in Fig. 8.



mass of Sun = 1.99×10^{30} kg

mass of Earth = 5.98×10^{24} kg

Earth-Sun distance = 1.50×10^{11} m

Fig. 8

(i) Calculate, using the data given,

1. the pull of the Earth on the satellite,

2. the pull of the Sun on the satellite. [3]

(ii) Using Fig. 8 as a guide, draw a sketch to show the relative positions of the Earth, the Sun and the satellite. On your sketch draw arrows to represent the two forces acting on the satellite. Label the arrows with the magnitude of the forces. [2]

(iii) Calculate

1. the magnitude and direction of the resultant force on the satellite,

2. the acceleration of the satellite. [3]

(iv) The satellite is in a circular orbit around *the Sun*. Calculate the angular velocity of the satellite. [3]

(v) Using your answer to *(a)* (ii) describe the motion of the satellite relative to the Earth. Suggest why this orbit around the Sun is preferable to a satellite orbit around the Earth. [3]

(vi) Suggest **two** disadvantages of having a satellite in this orbit. [2]

N2000/III/2