TOPIC 7 Circular Motion

1 Which of the following statements is correct for a particle moving in a horizontal circle with constant angular velocity?

- **A** The linear momentum is constant but the kinetic energy varies.
- **B** The kinetic energy is constant but the linear momentum varies.
- **C** Both kinetic energy and linear momentum are constant.
- **D** Both speed and linear velocity are constant.
- **E** Neither the linear momentum nor the kinetic energy is constant.

N76/II/5; J86/I/6; N93/I/6

2 The maximum safe speed of a car rounding an unbanked corner is 20 m s^{-1} when the road is dry. The maximum frictional force between the road surface and the wheels of the car is halved when the road is wet.

What is the maximum safe speed for the car to round the corner when the road is wet?

- **A** $\frac{20}{4}$ m s^{-1}
- **B** $\frac{20}{2\sqrt{2}}$ m s^{-1}
- **C** $\frac{20}{2}$ m s^{-1}
- **D** $\frac{20}{\sqrt{2}}$ m s^{-1}

J78/II/37; J97/I/8

3 An object of mass of 2 kg rotates at constant speed in a horizontal circle of radius 5 m. The time for one complete revolution is 3 s.

What is the magnitude of the resultant force acting on the object?

- **A** $\frac{4\pi^2}{9}$ N
- **B** $\frac{40\pi^2}{9}$ N
- **C** $\frac{100\pi^2}{9}$ N
- **D** $\frac{400\pi^2}{9}$ N

J79/II/1; J99/I/8

4 A body of mass m moves in a horizontal circle of radius r at constant speed v (Fig. 1). Which pair of values correctly gives (i) the work done by the centripetal force, (ii) the change in linear momentum of the body, when it moves from X to Y (where XY is a diameter)?

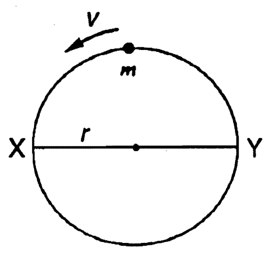
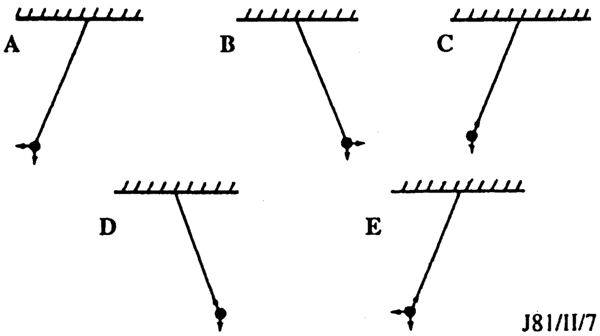


Fig. 1

	(i) work done	(ii) change in linear momentum
A	$2mv^2$	$2mv$
B	πmv^2	$2mv$
C	0	0
D	0	$2mv$
E	πmv^2	0

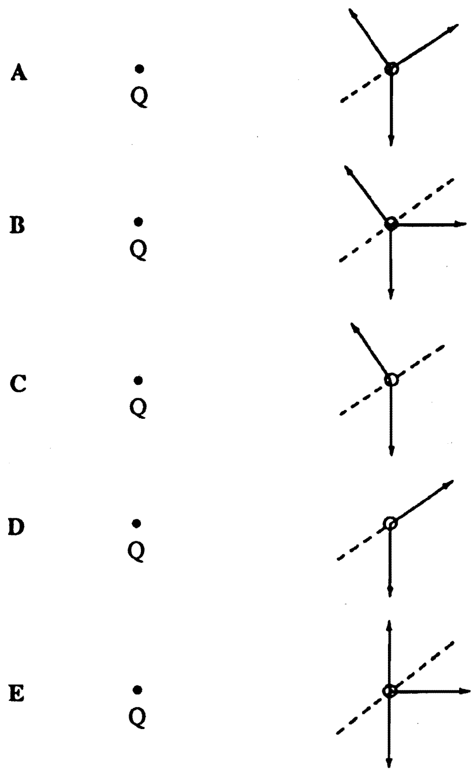
N79/II/7

5 A passenger is siting in a railway carriage facing in the direction in which the train in travelling. A pendulum hangs down in front of him from the carriage roof. The train travels along a circular arc bending to the right. Which one of the following diagrams shows the position of the pendulum as seen by the passenger and the directions of the forces acting on it?



J81/II/7

6 An aircraft is travelling at constant speed in a horizontal circle, centre Q. Each diagram below shows a tailview of the aircraft, the dotted line representing the line of the wings and the circle representing the centre of gravity of the aircraft. Which one of the diagrams correctly shows the forces acting on the aircraft?



N81/II/4

7 A car of mass m moving at a constant speed v passes over a humpback bridge of radius of curvature r. Given that the car remains in contact with the road, what is the net force R exerted by the car on the road when it is at the top of the bridge?

A $R = mg + mv^2/r$
B $R = mv^2/r$
C $R = mg - mv^2/r$
D $R = mg$
E $R = mv^2/r - mg$ J82/II/6; N85/I/4

8 A body moves in a circle with increasing angular velocity. At times t, the angles θ swept out by the body and its angular velocities ω are as follows:

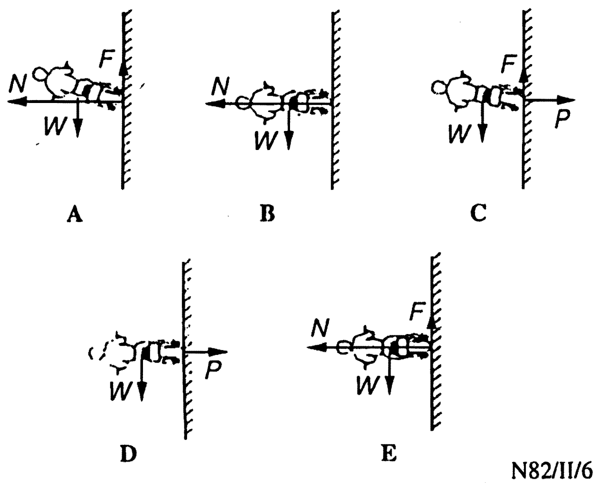
t/s	θ/rad	ω/rad s^{-1}
2	14	11
4	44	19
6	90	27
8	152	35

The angular acceleration of the body

A gradually decreases and is 4.5 rad s^{-2} when $t = 6$ s.
B is constant at 4 rad s^{-2}.
C is constant at 8 rad s^{-2}.
D increases at a constant rate and is 15 rad s^{-2} when $t = 6$ s.
E gradually increases and is 4.5 rad s^{-2} when $t = 6$ s.
J82/II/7

9 A motor-cyclist moving sufficiently fast may travel in a horizontal circle on the inside surface of a vertical cylindrical wall (a "wall of death"). Which one of the diagrams below, which are vertical sections, correctly shows the orientation of the motor-cycle and the rider at any instant, and the directions of the forces acting?

(In the diagrams, F is a fractional force, W is the total weight of rider and motor-cycle, N is the normal reaction of the surface on the motor-cycle, and P is an outward force.)



N82/II/6

10 A wheel, mass m, of which the centre of mass G is not at its centre is mounted on bearings with its axle horizontal and it rotates about its centre O with constant angular velocity ω as shown (Fig. 2).

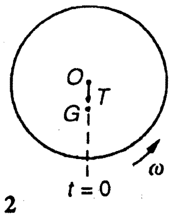
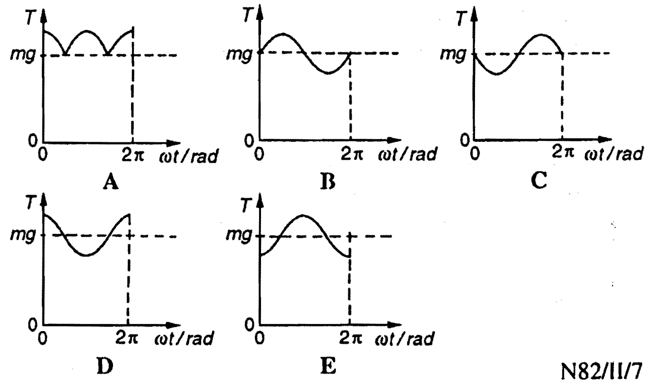


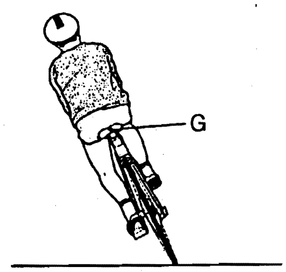
Fig. 2

Which one of the following graphs best illustrates the variation of the downward thrust T on the bearings as the wheel rotates through one revolution?

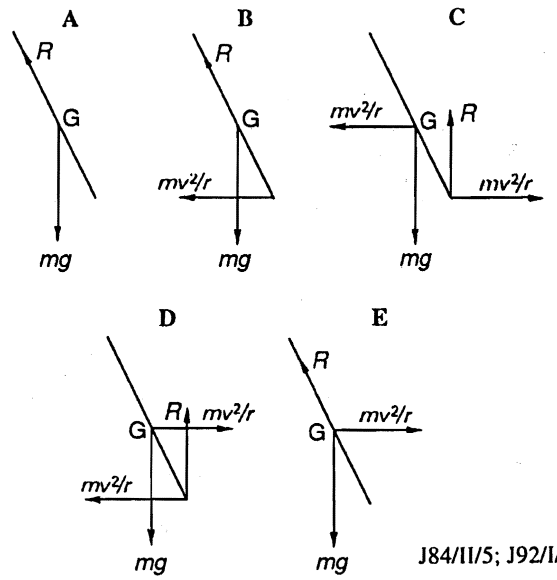


N82/II/7

11 The diagram represents a cyclist making a left turn on a rough road surface at constant speed v, as viewed from behind. The total mass of the bicycle and rider is m and their combined centre of gravity is at G.



If R is the resultant force of the normal reaction and the frictional force, which vector diagram represents the directions of the forces acting on the bicycle and its rider?



J84/II/5; J92/I/7

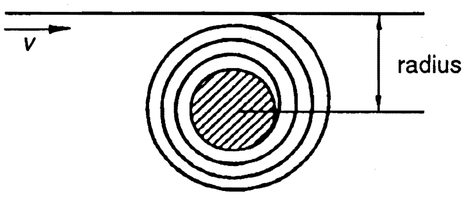
12 A communications satellite of mass m moves at constant angular speed ω in a circular orbit of radius r about the Earth's centre of mass.

What is the work done on the satellite in one revolution?

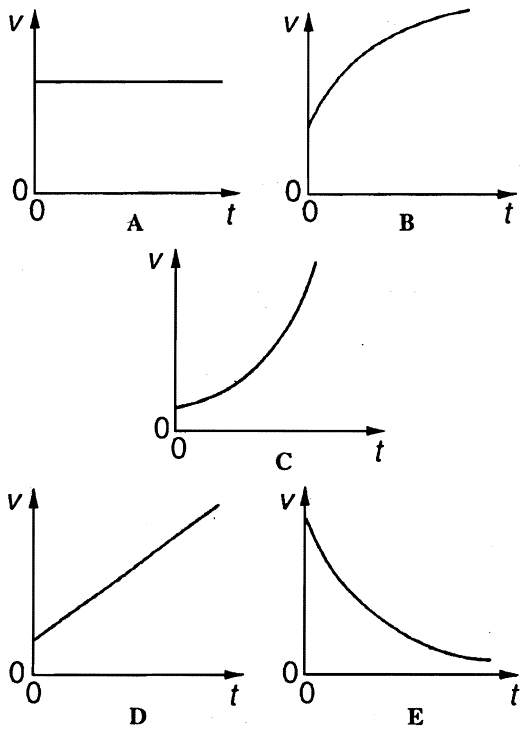
A zero
B $2\pi mr^2\omega^2$
C $\pi mr^3\omega^2$
D $mr^2\omega^2$
E $\frac{1}{2}mr^2\omega^2$

N84/II/6; J93/I/7

13 A straight length of tape winds on to a roll rotating about a fixed axis with constant angular velocity, the radius of the roll increasing at a steady rate.



Which one of the graphs below correctly shows how the *speed* v at which the tape moves towards the roll varies with time?



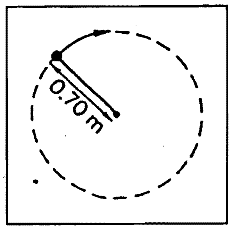
N86/I/5

14 The minute hand of a large clock is 3.0 m long. What is its mean angular speed?

A 1.4×10^{-4} rad s^{-1}
B 1.7×10^{-3} rad s^{-1}
C 5.2×10^{-3} rad s^{-1}
D 1.0×10^{-1} rad s^{-1}
E 3.0×10^{-1} rad s^{-1}

J87/I/7

***15** A mass of 0.050 kg is attached to one end of a piece of elastic of unstretched length 0.50 m. The force constant of the elastic (i.e. the force required to produce unit extension) is 40 N m^{-1}. The mass is rotated steadily on a smooth table in a horizontal circle of radius 0.70 m as shown.



What is the approximate speed of the mass?

A 11 m s^{-1}
B 15 m s^{-1}
C 20 m s^{-1}
D 24 m s^{-1}
E 28 m s^{-1}

J87/I/9

16 An artificial satellite travels in a circular orbit about the Earth. Its rocket engine is then fired and produces a force on the satellite exactly equal and opposite to that exerted by the Earth's gravitational field.

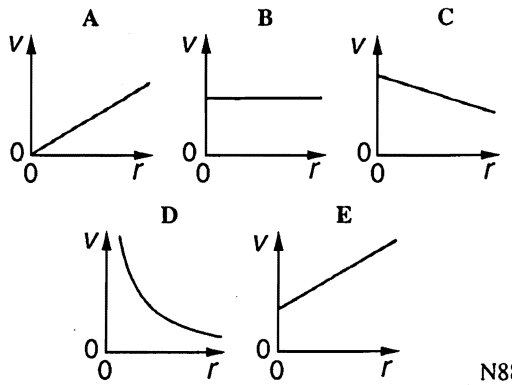
The satellite would then start to move

A along a spiral path towards the Earth's surface.
B along the line joining it to the centre of the Earth (i.e. radially).
C along a tangent to the orbit.
D in a circular orbit with a longer period.
E in a circular orbit with a shorter period.

J88/I/7

17 A record on a turntable is rotating at a constant number of revolutions per second.

Which graph best represents the relation between the speed v of a point on the record and his distance r from the centre of rotation?



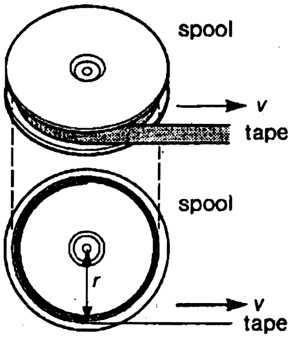
N88/I/6

18 When the aircraft Concorde is moving in a horizontal plane at a constant speed of 650 ms^{-1}, its turning circle has a radius of 80 km. What is the ratio of the centripetal force to the weight of the aircraft?

A 8.3×10^{-4}
B 0.54
C 1.9
D 52
E 540

J90/I/9

19 In a tape cassette, the tape leaves one spool at a constant speed v and at a variable distance r from the centre.



The angular velocity of the spool

A is proportional to $\frac{1}{r^2}$.

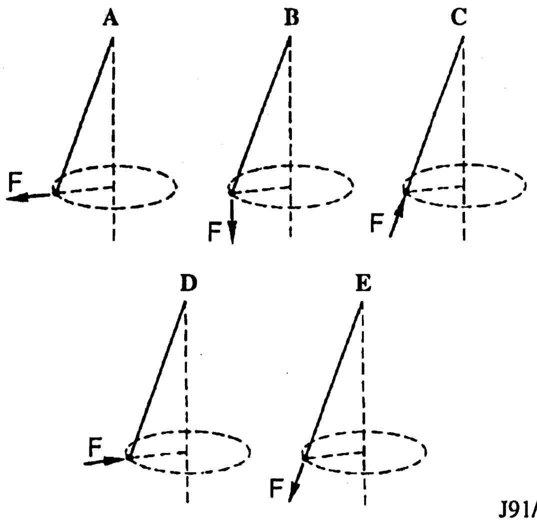
B is proportional to $\frac{1}{r}$.

C is proportional to r.

D does not depend on r. N90/I/10; J95/I/7

20 A mass on the end of a string is set in motion so that it describes a circle in a horizontal plane.

Which diagram shows the direction of the resultant force acting on the mass at an instant in its motion?



J91/I/8

21 A particle travels in uniform circular motion.

Which of the following correctly describes the linear velocity, angular velocity and linear acceleration of the particle?

	linear velocity	*angular velocity*	*linear acceleration*
A	constant	constant	varying
B	constant	constant	zero
C	constant	varying	constant
D	varying	constant	varying
E	varying	varying	constant

N91/I/9

22 A body moving in a circular path of radius r has tangential acceleration a_t, and centripetal acceleration a_c.

If the body is moving at constant speed v, what are the magnitudes of a_t and a_c?

	tangential acceleration a_t	*centripetal acceleration* a_c
A	rv^2	0
B	v^2/r	0
C	0	rv^2
D	0	v^2/r

J94/I/8

23 An object travels at constant speed around a circle of radius 1.0 m in 1.0 s. What is the magnitude of its acceleration?

A zero **C** 2π m s^{-2}

B 1.0 m s^{-2} **D** $4\pi^2$ m s^{-2}

N95/I/8

24 A satellite moves at constant speed in a circular orbit about the Earth.

Which statement about the momentum and kinetic energy of the satellite is correct?

	momentum	*kinetic energy*
A	constant	changing
B	constant	constant
C	changing	changing
D	changing	constant

J96/I/8

25 A disc is rotating about an axis through its centre and perpendicular to its plane. A point P on the disc is twice as far from the axis as a point Q.

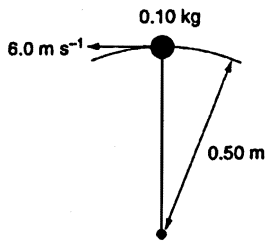
At a given instant what is the value of $\frac{\text{the linear velocity of P}}{\text{the linear velocity of Q}}$?

A 4 **B** 2 **C** ½ **D** ¼

N96/I/9

26 A ball of mass 0.10 kg is attached to a string and swung in a vertical circle of radius 0.50 m, as shown. Its speed at the top of the circle is 6.0 m s^{-1}.

[Take g as 10 m s^{-2}.]



What is the tension in the string at this moment?

A 1.0 N **C** 7.2 N

B 6.2 N **D** 8.2 N

N97/I/8

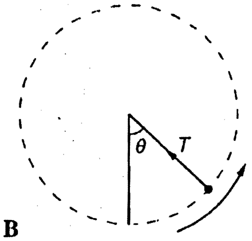
27 Satellites are in circular orbit around the Earth.

What is the relationship between the radii r of their orbits and their speeds v?

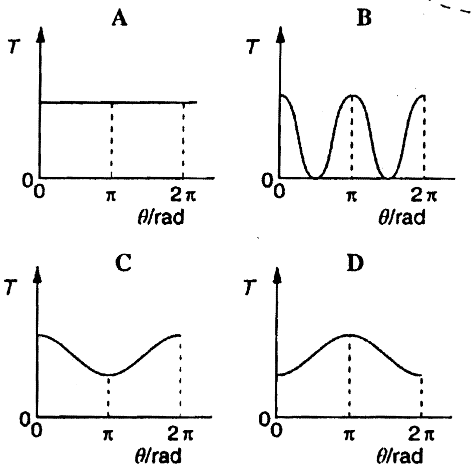
A $v \propto r^2$ **B** $v \propto r$ **C** $v^2 \propto \frac{1}{r}$ **D** $v \propto \frac{1}{r^2}$

N98/I/8

28 An object is fixed to one end of a light rod which rotates in a vertical circle at constant speed.



Which graph could represent the variation with angle θ of the tension T in the rod?



N99/I/8

29 A stone of mass m is attached to a string of length r, which will break if the tension in it exceeds T_{max}. The stone is whirled in a vertical circle.

(a) Draw diagrams showing the forces acting on the stone when it is (i) at the top, (ii) at the bottom, of the circle.

The angular speed is increased very slowly.

(b) For what position of the stone, relative to the axis of rotation, is the string most likely to break?

(c) What will be the angular speed when this occurs?

J77/I/1

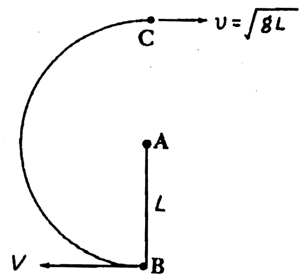
30 Find the angular velocity of the seconds hand of a watch.

N78/I/1

31 A man stands at the Earth's equator. Find *(a)* his angular velocity, *(b)* his linear speed, *(c)* his acceleration, due to the rotation of the Earth about its axis.

[1 day = 8.6×10^4 s; radius of Earth = 6.4×10^6 m.] J81/I/2

32 A particle is suspended from a point A by an inextensible string of length L. It is projected from B with a velocity V, perpendicular to AB, which is just sufficient for it to reach the point C.



(a) Show that, if the string is just to be taut when the particle reaches C, its speed there is $\sqrt{(gL)}$.

(b) Find the speed V with which the particle should be projected from B.

N81/I/1

33 The reading of a speedometer fitted to the front wheel of a bicycle is directly proportional to the angular velocity of the wheel. A certain speedometer is correctly calibrated for use with a wheel of diameter 66 cm but, by mistake, is fitted to a 60 cm wheel. Explain whether the indicated linear speed would be greater or less than the actual speed and find the percentage error in the readings. J84/I/1

34 The sketch (Fig. 3) shows an aircraft A performing a horizontal circular turn about the point O. Obtain an expression for the acceleration of the aircraft as it moves in the circle in terms only of its weight W, the lift L and the acceleration of free fall g.

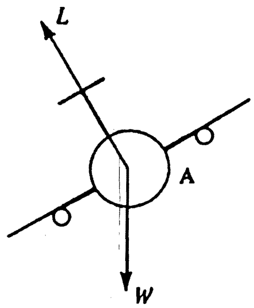
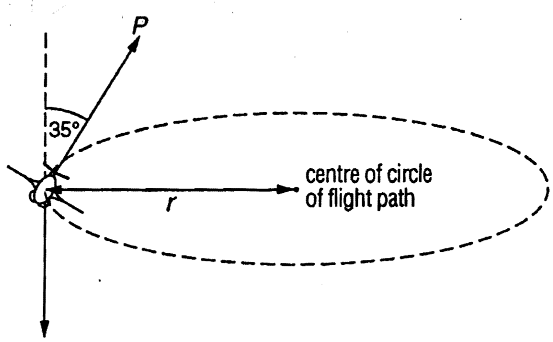


Fig. 3

N84/I/2

35 A record is played at 45 revolutions per minutes, and then at $33\frac{1}{3}$ revolutions per minute. Find the ratio of the centripetal accelerations of a point on the rim of the record. N86/II/2

36 An aircraft flies with its wings tilted as shown in Fig. 4 in order to fly in a horizontal circle of radius r. The aircraft has mass 4.00×10^4 kg and has a constant speed of 250 m s^{-1}.



$W = 3.92 \times 10^5$ N **Fig. 4**

With the aircraft flying in this way, two forces acting on the aircraft in the vertical plane are the force P acting at an angle of 35° to the vertical and the weight W.

(a) State the vertical component of P for horizontal flight. [1]

(b) Calculate P. [2]

(c) Calculate the horizontal component of P. [1]

(d) Use Newton's second law to determine the acceleration of the aircraft towards the centre of the circle. [2]

(e) Calculate the radius r of the path of the aircraft's flight. [2]

J2000/II/3

Long Questions

37 Use Newton's laws of motion to explain why a body moving with uniform speed in a circle must experience a force towards the centre of the circle.

An aircraft of mass 1.0×10^4 kg is travelling at a constant speed of 0.2 km s^{-1} in a horizontal circle of radius 1.5 km.

(a) What is the angularly velocity of the aircraft?

(b) Show on a sketch the forces acting on the aircraft in the vertical plane containing the aircraft and the centre of the circle. Find the magnitude and direction of their resultant.

(c) Explain why a force is exerted on a passenger by the aircraft. In what direction does this force act?

J78/I/14

38 An aircraft is travelling at a constant speed of 180 m s^{-1} in a horizontal circle of radius 20 km. A plumbline, attached to the roof of the cabin, settles at an angle ϕ to the true vertical while the aircraft is turning.

(i) Find the centripetal acceleration of the aircraft.

(ii) Name the forces which act on the bob of the plumbline, and draw a labelled diagram to show the directions of these forces, and of their resultant. (Indicate the centre of the circle on your sketch.)

(iii) Find the angle ϕ.

(iv) Show by means of a simple sketch of the cross-section of the aircraft and its cabin how the plumbline is oriented with respect to the aircraft.

J85/II/8 (part)

39 Astronomical observations show that during the period from 1870 to 1900 the length of the day increased by about 6×10^{-3} s.

(a) Express this increase as a fraction of a day.
(1 day = 8.64×10^4 s.)

(b) To what fractional change in the Earth's angular velocity does this correspond?

N85/II/9 (part)

40 *(a)* A body of mass m travels at constant speed in a circular path of radius r. It takes time T to complete one revolution.

(i) Write down expressions in terms of m, r and T for the speed, the acceleration, the angular velocity, the kinetic energy, the momentum and the angular momentum of the body. [6]

(ii) Which of these quantities change during a revolution and which remain constant? [3]

(iii) On a sketch show the directions of the acceleration and the momentum at a particular instant. [2]

J88/II/9 (part)

41 Each blade on a turbine wheel is attached separately to a small section of the rim of the wheel as shown in Fig. 5.

The blades are small and each behaves like a point mass of 0.72 kg at a distance of 0.65 m from the axis of rotation.

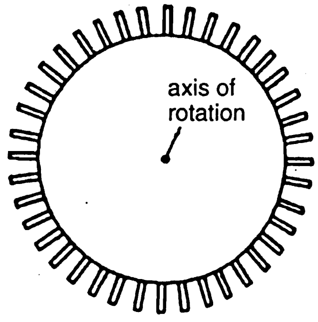


Fig. 5

The wheel is tested by spinning it at high speed. The plane of the wheel is kept horizontal with the axis of rotation vertical. It is found that blades break off at angular velocities greater than 540 rad s^{-1}.

(a) Outline an experimental method for measuring the angular velocity of the wheel. [3]

(b) Calculate the linear speed of the blades when the angular velocity is 540 rads^{-1}. What is the corresponding centripetal acceleration? [4]

(c) Use Newton's laws to explain why a blade might break off at high angular velocities. [4]

(d) Calculate the minimum radial force required to pull a blade off the wheel. [2]

N89/II/9 (part)

42 *(a)* A body moves at constant angular velocity ω in a circle of radius r. State its acceleration. [2]

(b) In a ride at an entertainments park, two people, each of mass 80 kg, sit in cages which travel at constant speed in a vertical circle of radius 8.0 m as shown in Fig. 6. Each revolution takes 4.2 s. When a cage is at the top of the circle (position A) the person in it is upside down.

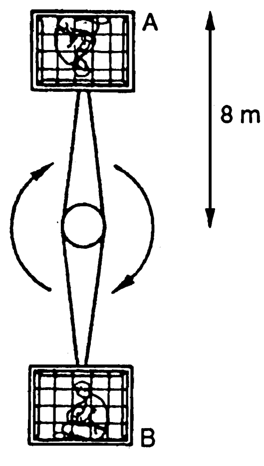


Fig. 6

For the person in cage A calculate the magnitudes of

(i) the angular velocity,

(ii) the linear speed,

(iii) the centripetal acceleration. [4]

(c) (i) Draw a vector diagram to show the directions of the following forces acting on the person in cage A in Fig. 6;

the weight W of the person,
the force F exerted by the cage on the person.

(ii) Draw the corresponding diagram for the person at the bottom of the circle (position **B**).

(iii) What must be the value of the resultant of these two forces at both **A** and **B**?

(iv) Explain why the person remains on the floor of the cage at the top of the circle.

(v) State the position of the cage at which the force it exerts on the person has its maximum value. Calculate the magnitude of this force. [11]

(d) Draw a vector diagram showing W, F and their resultant when the line joining the cages is horizontal. Numerical values are not required for this part of the question, but the force vectors should be drawn so that they have approximately correct relative sizes. [3]

N90/III/2

43 *(a)* State what is meant by angular velocity. [2]

(b) A stone is tied to one end of a cord and then made to rotate in a horizontal circle about a point C with the cord horizontal, as shown in Fig. 7.

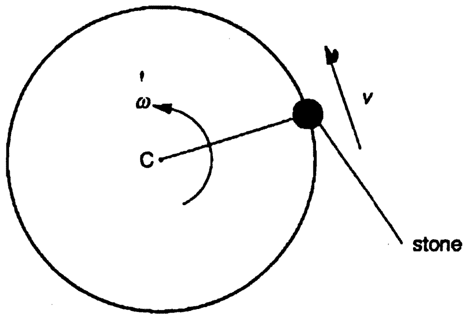


Fig. 7

The stone has speed v and angular velocity ω about **C**.

(i) Write down a relation between the speed v, the length r of the cord and the angular velocity ω.

(ii) Explain how v can be made to vary when ω is constant.

(iii) Explain why there needs to be a tension in the cord to maintain the circular motion.

(iv) Write down an expression for the acceleration of the stone in terms of v and r. Hence, if the stone has mass m, show that the tension T in the cord is given by

$$T = mv\omega.$$ [8]

(c) On one particular ride in an amusement park, passengers 'loop-the-loop' in a vertical circle, as illustrated in Fig. 8.

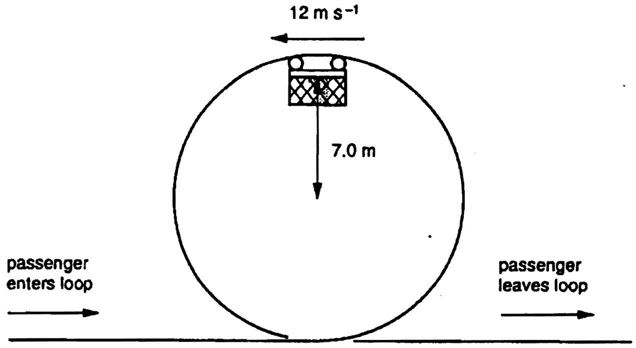


Fig. 8

The loop has a radius of 7.0 m and a passenger, mass 60 kg, is travelling at 12 m s^{-1} when at the highest point of the loop. Assume that frictional forces may be neglected.

(i) Calculate, for the passenger when at the highest point,

(1) the centripetal acceleration,

(2) the force the seat exerts on the passenger.

(ii) The passenger now moves round and descends to the bottom of the loop. Calculate

(1) the change in potential energy of the passenger in moving from the top of the loop to the bottom,

(2) the speed of the passenger on leaving the loop.

(iii) Operators of this ride must ensure that the speed at which the passengers enter the loop is above a certain minimum value. Suggest a reason for this. [10]

J94/III/1

44 *(a)* Describe qualitatively how it is that a body which is travelling in a circle with uniform speed has acceleration. Show on a diagram the direction of this acceleration in relation to the direction in which the body is travelling. [4]

**(b)* State the equation for the force of attraction F between two point charges of equal magnitude, $+q$ and $-q$, when placed a distance r apart in a vacuum. [2]

**(c)* There is a sub-atomic particle of the same mass as the electron but with a positive charge equal in magnitude to the charge of the electron. This particle is called a positron. Calculate the force between an electron and a positron when they are separated by a distance of 1.30×10^{-10} m. [3]

(d) It is possible for an electron and a positron to move in a circular orbit as shown in Fig. 9. This system is called positronium and it is very unstable.

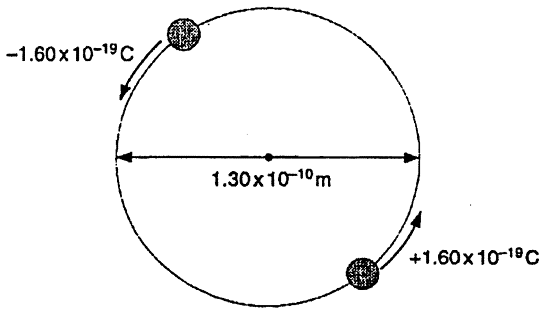


Fig. 9

Using your answer to part *(c)* calculate

(i) the acceleration of the electron (mass 9.11×10^{-31} kg),

(ii) the radius of the orbit of the electron,

(iii) the speed of the electron in its orbit,

(iv) the time taken for one revolution of the electron. [8]

(e) Suggest, with a reason, what will happen to the two particles in *(d)* if their total energy is reduced. [3]

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