TOPIC 12 Oscillations

Simple Harmonic Motion

1 A small mass executes s.h.m. about a point O with amplitude a and period T. Its displacement from O at time $T/8$ after passing through O is

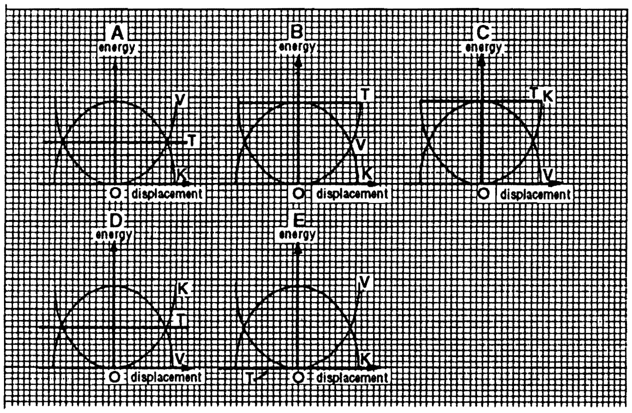
A $a/8$ D $a/\sqrt{2}$

B $a/2\sqrt{2}$ E $\frac{(2\sqrt{2})a}{3}$

C $a/2$

J76/II/9

2 Which one of the following graphs best represents the relation between the kinetic energy K, the potential energy V and the total energy T of a particle moving in a straight line with simple harmonic motion?



J76/II/10

3 When the length of a simple pendulum is doubled, the ratio of the new frequency to the old frequency is

A 2 B $\sqrt{2}$ C $\frac{1}{\sqrt{2}}$ D $\frac{1}{2}$ E $\frac{1}{4}$

N76/II/9

4 When a particle performs simple harmonic motion the velocity leads the displacement by a phase angle of

A $\pi/4$ rad D π rad

B $\pi/2$ rad E zero

C $3\pi/4$ rad

J77/II/9

5 A point moves with s.h.m. along an x-axis according to the equation

$$\frac{d^2x}{dt^2} + Ax = 0.$$

The period of this motion is

A $\frac{\sqrt{A}}{2\pi}$ D $\frac{2\pi}{\sqrt{A}}$

B $\frac{\sqrt{A}}{\pi}$ E $\frac{\pi}{2\sqrt{A}}$

C $\frac{\pi}{\sqrt{A}}$

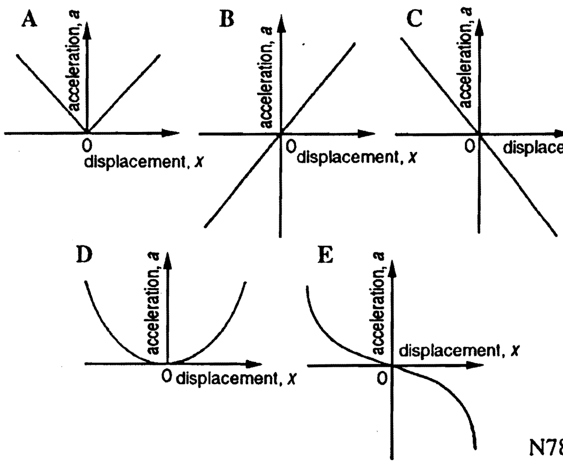
N77/II/10

6 In order to check the speed of a camera shutter, the camera was used to photograph the bob of a simple pendulum moving in front of a horizontal scale. The extreme positions of the bob were at the 600 mm and 700 mm marks. The photograph showed that while the shutter was open the bob moved from the 650 mm mark to the 675 mm mark. If the period of the pendulum was 2 s, the shutter remained open for

A 1 s B $^1/_2$ s C $^1/_3$ s D $^1/_4$ s E $^1/_6$ s

J78/II/10

7 Which one of the following sketch graphs best represents the relation between the acceleration a of a body executing a simple harmonic motion and the displacement x of the body from the centre of its path?



N78/II/10

8 A particle performs simple harmonic motion of amplitude 2.0×10^{-3} m and period 0.10 s. It maximum speed is approximately

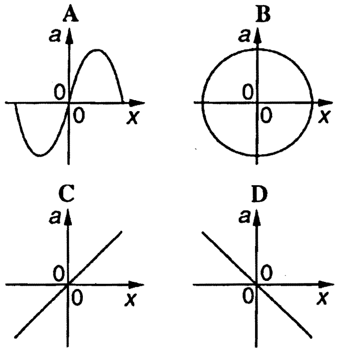
A 3.2×10^{-5} m s^{-1} D 1.3×10^{-1} m s^{-1}

B 2.0×10^{-4} m s^{-1} E 5.0×10^{4} m s^{-1}

C 2.0×10^{-2} m s^{-1}

J79/II/9

9 Which graph shows the relationship between the acceleration a and the displacement x of a particle performing simple harmonic motion?



J79/II/10; J93/I/8; J96/I/9

10 The displacement x of a particle at a time t is given by

$$x\,/\,\text{m} = 5 \sin (2t\,/\,\text{s.})$$

A simple pendulum has the same period as the particle when the length of the pendulum is

A 10.0 m
B 5.0 m
C 2.5 m [Take g as 10.0 m s^{-2}.]
D 2.0 m
E 0.4 m

N79/II/9

11 When a particle of mass m is suspended by a string of unstretched length a, the string extends to a total length l. The particle is pulled down a small distance b and released. It then oscillates in simple harmonic motion, the equation describing the motion being $\ddot{x} + \omega^2 x = 0$, where x is displacement and ω^2 is

A $mg\,/\,(l-a)$ **D** $gb\,/\,(l-a)$
B $mga\,/\,(l-a)$ **E** $g\,/\,(l-b)$
C $g\,/\,(l-a)$

N79/II/10

12 A body in simple harmonic motion makes n complete oscillations in one second. The angular frequency of this motion is

A n rad s^{-1} **D** $2\pi/n$ rad s^{-1}
B $1/n$ rad s^{-1} **E** $n/2\pi$ rad s^{-1}
C $2\pi n$ rad s^{-1}

J80/II/13; J83/II/9

13 A particle rotates clockwise in a horizontal circle of radius r with a constant angular velocity ω as shown in (Fig. 1). The particle is at S at time zero and at P at time t. Q represents the projection of point P on to the diameter through S. Measured with respect to the origin O, the displacement, linear velocity and linear acceleration of Q in the direction OS are y, v and a respectively.

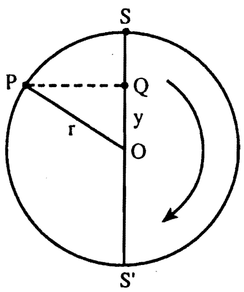


Fig. 1

Which one of the following sets of expressions is correct?

A $y = r\cos \omega t$; $v = -r\omega \sin \omega t$; $a = rw^2 \cos \omega t$
B $y = r\cos \omega t$; $v = -r\omega \sin \omega t$; $a = -rw^2 \cos \omega t$
C $y = r\cos \omega t$; $v = -r\omega \cos \omega t$; $a = -rw^2 \sin \omega t$
D $y = r\sin \omega t$; $v = -r\omega \cos \omega t$; $a = -rw^2 \sin \omega t$
E $y = r\sin \omega t$; $v = r\omega \cos \omega t$; $a = rw^2 \sin \omega t$

J80/II/14

14 A mass m on a smooth horizontal table is attached by two light springs to two fixed supports as shown below (Fig. 2). The mass executes linear simple harmonic motion of amplitude a and period T

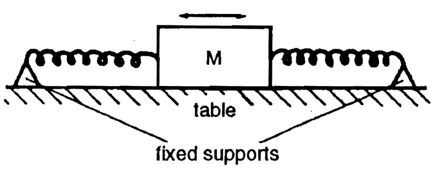


Fig. 2

The energy associated with this simple harmonic motion is

A $2\pi ma^2\,/\,T^2$ **D** $2\pi^2 ma^2\,/\,T^2$
B $2\pi m^2 a^2\,/\,T$ **E** $4\pi^2 ma\,/\,T^2$
C $\pi^2 ma^2\,/\,T^2$

J81/II/16

15 Values of the acceleration $\ddot{x}$ of a particle moving in simple harmonic motion as a function of its displacement x are given in the table below.

$\ddot{x}$ / mm s^{-2}	16	8	0	–8	–16
x / mm	–4	–2	0	2	4

The period of the motion is

A $1/\pi$ s **D** 2 s
B $2/\pi$ s **E** π s
C $\pi/2$ s

N81/II/11; J86/I/7

16 A particle moves so that its potential energy U varies with the square of its displacement r from the origin, i.e. $U \propto r^2$ (see Fig. 3).

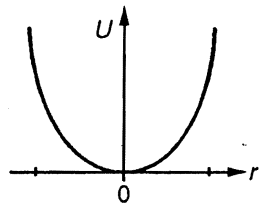
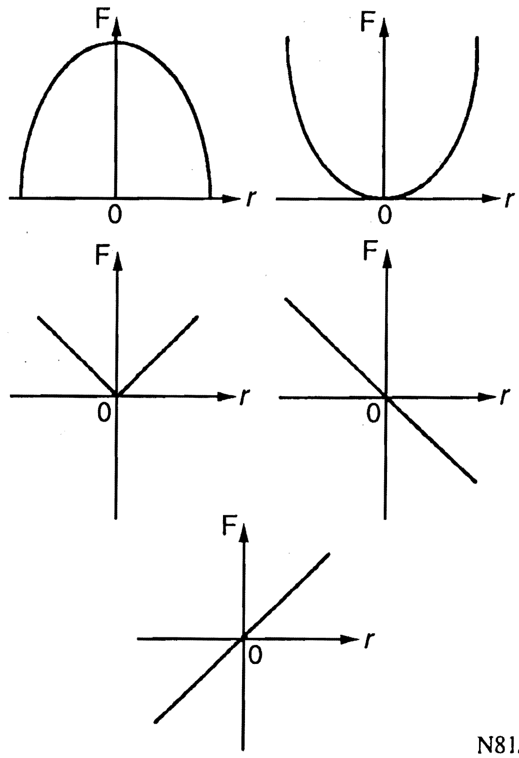


Fig. 3

Which one of the following graphs best represents the way in which the force F acting on the particle in the direction of increasing r depends on r?



N81/II/14

17 A trolley of mass 2 kg with free-running wheels is attached to two fixed points P and Q by two springs under tension as shown in Fig. 4 below.

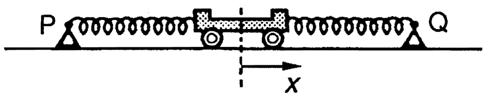


Fig. 4

The trolley is displaced a small distance (0.05 m) towards Q by a force of 10 N and is then released. The equation of the subsequent motion is $\ddot{x} = -\omega^2 x$, where x is the displacement from the equilibrium position. What is the constant ω^2?

A 0.25 rad^2 s^{-2}
B 1.0 rad^2 s^{-2}
C 4.0 rad^2 s^{-2}
B 100 rad^2 s^{-2}
E 400 rad^2 s^{-2}

J82/II/9

18 Fig. 5 *(a)* and *(b)* below show the displacement x and the acceleration a of a body vary with time when it is oscillating with simple harmonic motion.

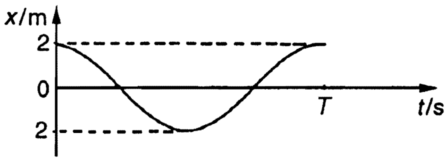


Fig. 5*(a)*

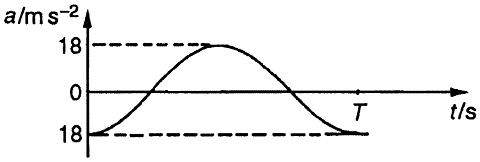


Fig. 5*(b)*

What is the value of T?

A $\pi/9$
B $2\pi/9$
C $2\pi/3$
D 2π
E 6π

J82/II/10

19 A particle of mass 4 kg moves with simple harmonic motion and its potential energy U varies with position x as shown in Fig. 6 below.

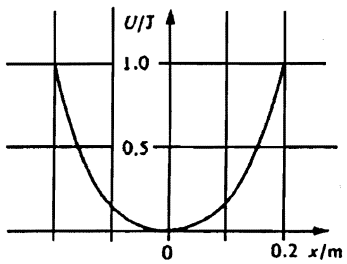


Fig. 6

What is the period of oscillation of the mass?

A $\dfrac{2\pi}{25}$ s
B $\dfrac{\pi\sqrt{2}}{5}$ s
C $\dfrac{8\pi}{25}$ s
D $\dfrac{4\pi}{5}$ s
E $\dfrac{2\pi\sqrt{2}}{5}$ s

N82/II/9

20 A particle moves such that its acceleration a is related to its displacement x from a fixed point as shown in the graph below (Fig. 7).

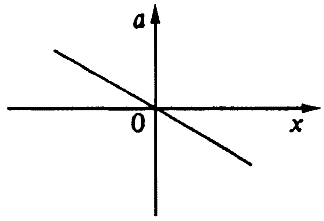
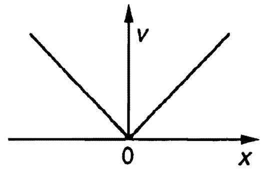


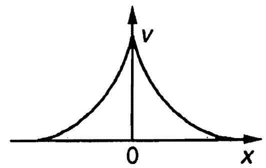
Fig. 7

Which one of the following best illustrates how the speed v varies with x?

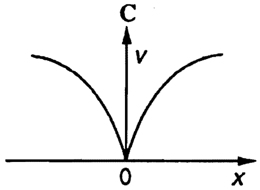
A



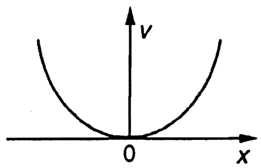
B



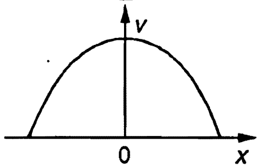
C



D



E



J83/II/8

21 All three systems represented as p, q and r in the diagrams below (Fig. 9) show simple harmonic motion.

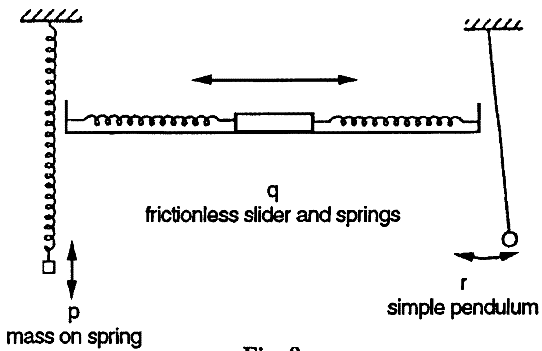


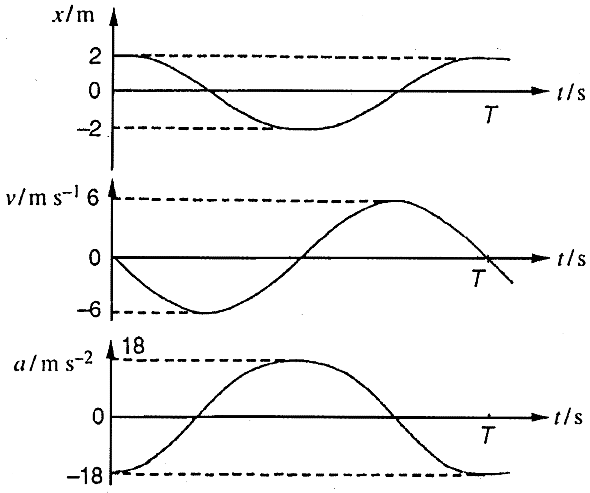
Fig. 9

In which system will the period be independent of the mass of the body?

A p only
B q only
C r only
D p and r only
E p, q and r

J84/II/9

22 The graphs below show how the displacement x, velocity v and the acceleration a of a body vary with time t when it is oscillating with simple harmonic motion.



What is the value of T?

A	$\pi / 9$ s	**D**	$2\pi / 3$ s
B	$2\pi / 9$ s	**E**	2π s
E	$\pi / 3$ s		

N84/II/9

23 In which of the following lists are all three quantities constant when a particle moves in undamped simple harmonic motion?

A	acceleration	force	total energy
B	amplitude	angular frequency	acceleration
C	angular frequency	acceleration	force
D	force	total energy	amplitude
E	total energy	amplitude	angular frequency

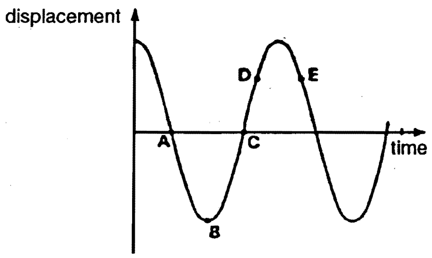
J85/I/8; J92/I/9

***24** The ionosphere contains free electrons. What is the amplitude of oscillation of these electrons when subject to a 200 kHz electromagnetic wave in which the oscillations of electric field have amplitude 5×10^{-3} V m^{-1}?

A	3.2×10^{-15} m	**D**	5.6×10^{-4} m
B	4.0×10^{-9} m	**E**	2.2×10^{-2} m
C	2.5×10^{-8} m		

J85/I/13

25 The diagram below shows a displacement-time graph of a body performing simple harmonic motion.



At which one of the points, **A, B, C, D,** or **E,** is the body travelling and accelerating in the same direction? N85/I/7

26 What is the frequency of a simple harmonic motion in which the acceleration a is related to the displacement x by the equation $a = -\omega^2 x$?

A	ω	**D**	$\dfrac{2\pi}{\omega}$
B	$\dfrac{1}{\omega}$	**E**	$\dfrac{\omega}{2\pi}$
C	$2\pi\omega$		

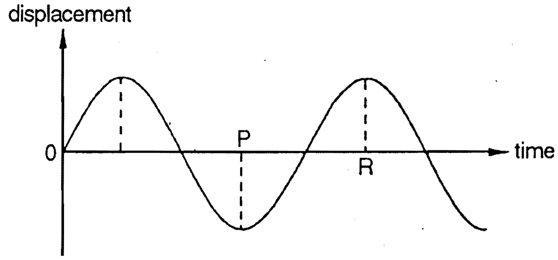
N86/I/7

27 A simple pendulum suspended from the ceiling of a stationary lift has period T_0. When the life descends at steady speed the period is T_1, and when it descends with constant downward acceleration the period is T_2. Which one of the following is correct?

A	$T_0 = T_1 = T_2$	**D**	$T_0 < T_1 < T_2$
B	$T_0 = T_1 < T_2$	**E**	$T_0 > T_1 > T_2$
C	$T_0 = T_1 > T_2$		

N86/I/8

28 In the diagram below, the displacement of an oscillating particle is plotted against time.



What does the length PR on the time axis represent?

A	half the period	**D**	twice the frequency
B	twice the frequency	**E**	half the wavelength
C	half the frequency		

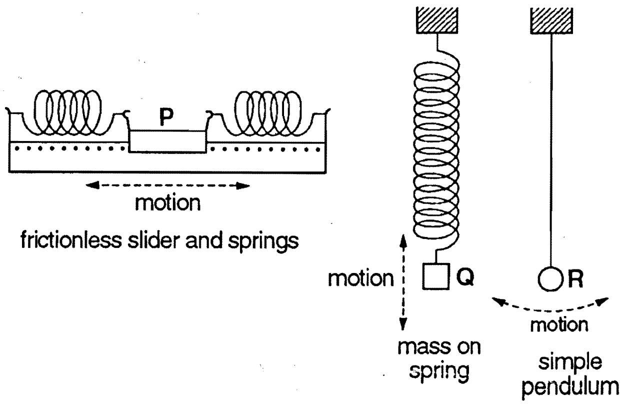
N86/I/9

29 A particle performs simple harmonic motion of amplitude 0.020 m and frequency 2.5 Hz. What is its maximum speed?

A	0.008 m s^{-1}	**D**	0.157 m s^{-1}
B	0.050 m s^{-1}	**E**	0.314 m s^{-1}
C	0.125 m s^{-1}		

J87/I/10

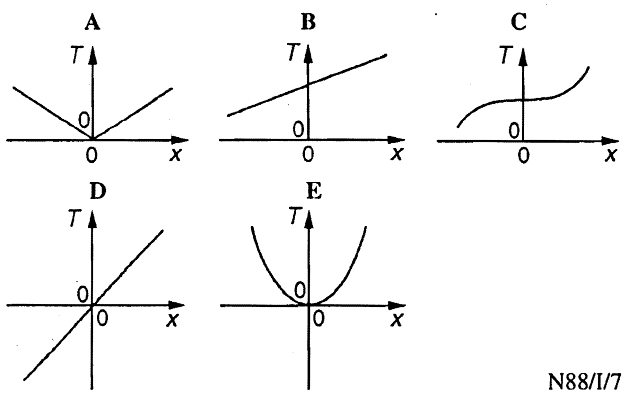
30 The three oscillating bodies, represented as **P**, **Q** and **R** in the diagrams, each show simple harmonic motion.



In which of these systems will the period increase if the mass of the body increases?

A P only
B Q only
C P and Q only
D Q and R only — N87/I/7; J94/I/9

31 A mass is hung from the free end of a light helical spring and then given a small displacement vertically downwards. Which graph best represents how T, the tension in the spring, varies with x, the displacement of the mass from the equilibrium position during the subsequent oscillations?



N88/I/7

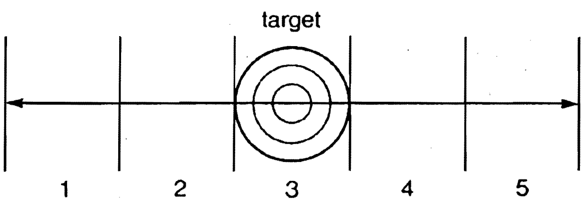
32 The acceleration of free fall on the Moon is one-sixth of that on the Earth. What would be the period on the Moon of a simple pendulum which has a period of 1 s on the Earth?

A $\frac{1}{6}$ s
B $\frac{1}{\sqrt{6}}$ s
C 1 s
D $\sqrt{6}$ s
E 6 s — J89/I/8

33 A body performing simple harmonic motion has a displacement x given by the equation $x = 30 \sin 50t$, where t is the time in seconds. What is the frequency of oscillation?

A 0.020 Hz
B 0.13 Hz
C 8.0 Hz
D 30 Hz
E 50 Hz — J90/I/10

34 In a fairground shooting game, a gun fires at a moving target. The gun fires by itself at random times. The player has to point the gun in a fixed direction, and the target moves from side to side with simple harmonic motion.

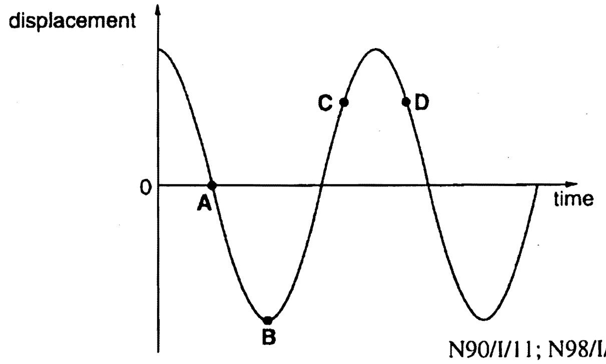


At which region should the player take a fixed aim in order to score the greatest number of hits?

A 3
B either 1 or 5
C either 2 or 4
D any of 1, 2, 3, 4 and 5 — J90/I/11; N95/I/9

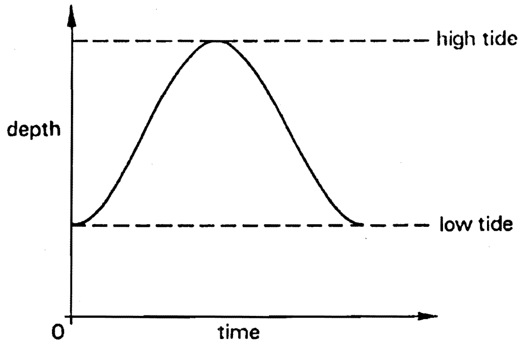
35 The diagram shows the graph of displacement against time for a body performing simple harmonic motion.

At which point are the velocity and acceleration in opposite directions?



N90/I/11; N98/I/9

36 The rise and fall of water in a harbour is simple harmonic. The depth varies between 1.0 m at low tide and 3.0 m at high tide. The time between successive low tides is 12 hours.



A boat, which requires a minimum depth of water of 1.5 m, approaches the harbour at low tide.

How long will the boat have to wait before entering?

A 0.5 hours
B 1.0 hours
C 1.5 hours
D 2.0 hours
E 2.5 hours — J91/I/9

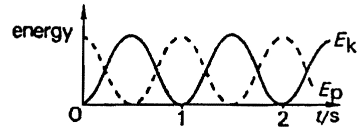
37 The cone of a loudspeaker sounding a note of frequency f executes simple harmonic motion of amplitude a.

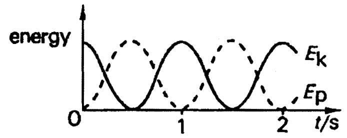
Which of the following expressions gives the maximum acceleration of the cone?

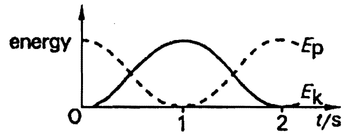
A fa
B $2\pi fa$
C $(fa)^2$
D $(2\pi f)^2 a$
E $(2\pi fa)^2$ — N91/I/10

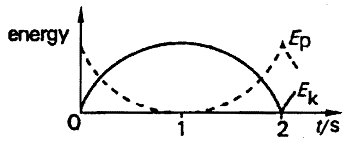
38 The bob of a simple pendulum of period 2s is given a small displacement and then released at time $t = 0$.

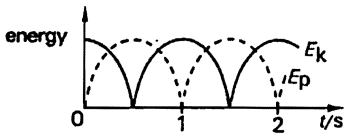
Which diagram shows the variations with time of the bob's kinetic energy E_k and its potential energy E_p?

A


B


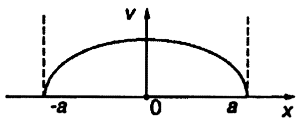
C


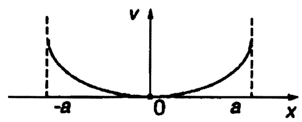
D


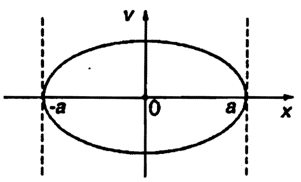
E


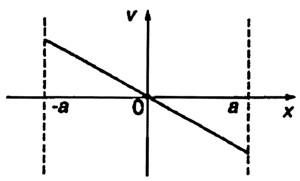
N92/I/9

39 Which graph best shows how the velocity v of an object performing simple harmonic motion of amplitude a varies with displacement x for one complete oscillation?

A


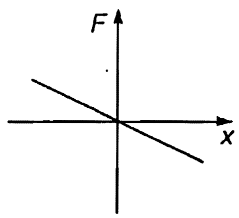
B


C


D


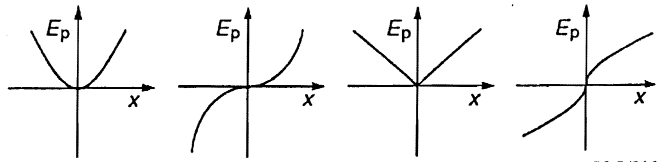
N94/I/9; N97/I/9

40 The resultant force F on a body moving in a straight line varies with displacement x from a fixed point as shown in the graph.



Which graph represents the variation with x of the potential energy E_p of the body?

A B C D



J95/I/6

41 A pendulum swings with the time period of 1 s at a place where the acceleration of free fall is g. If it were swung at another place where the acceleration of free fall is g', what would be the value of its new time period?

A $\left(\frac{g'}{g}\right)$ s

B $\left(\frac{g}{g'}\right)$ s

C $\sqrt{\left(\frac{g'}{g}\right)}$ s

D $\sqrt{\left(\frac{g}{g'}\right)}$ s

J95/I/9

42 Simple harmonic motion is **defined** as the motion of a particle such that

A its displacement x is always given by the expression $x = x_0 \sin\omega t$.

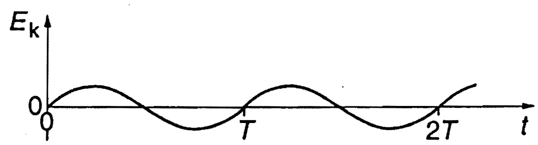
B its displacement x is related to its velocity v by the expression $v = \omega x$.

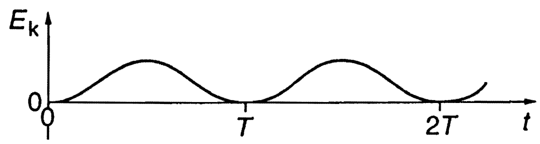
C its acceleration is always $\omega^2 x_0$ and is directed at right angles to its motion.

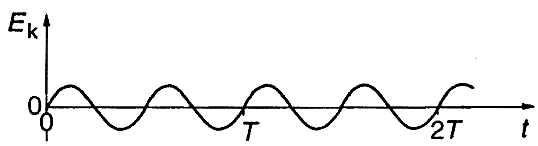
D its acceleration is proportional to, and in the opposite direction to, the displacement.

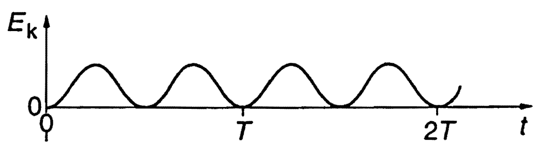
N96/I/10

43 Which graph correctly shows the variation with time t of kinetic energy E_k of an object undergoing simple harmonic motion of period T?

A


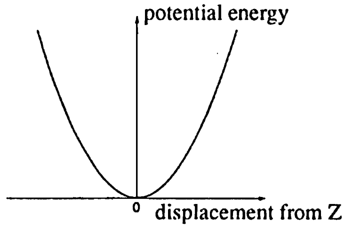
B


C


D


J97/I/9

44 The graph shows the way the potential energy of a body varies with its displacement from a point Z.



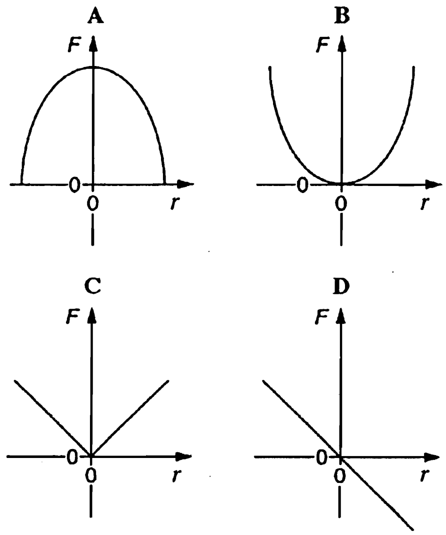
Which feature of the graph means that the force on the body is directed towards Z?

- **A** The graph is approximately linear for large displacements.
- **B** The graph passes through the origin.
- **C** The potential energy increases as the body moves further from Z.
- **D** The value of the potential energy is always positive.

J98/I/9

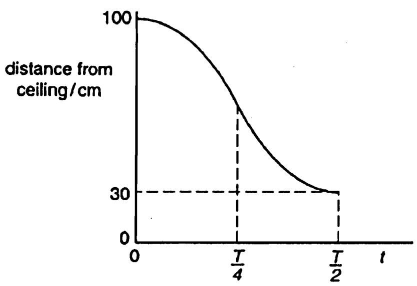
45 A resultant force F acts on a particle moving with simple harmonic motion.

Which graph shows the variation with displacement r of force F?



N99/I/9

46 A mass hanging from a spring suspended from the ceiling is pulled down and released. The mass then oscillates vertically with simple harmonic motion of period T. The graph shows how its distance from the ceiling varies with time t.



What can be deduced from this graph?

- **A** The amplitude of the oscillation is 70 cm.
- **B** The kinetic energy is a maximum at $t = \frac{T}{2}$.
- **C** The restoring force on the mass increases between $t = 0$ and $t = \frac{T}{4}$.
- **D** The speed is a maximum at $t = \frac{T}{4}$.

J2000/I/9

47 A particle moves with simple harmonic motion in a straight line with amplitude 0.05 m and period 12 s. Find *(a)* the maximum speed, *(b)* the maximum acceleration, of the particle. Write down the values of the constants P and Q in the equation

$$x\,/\,\mathrm{m} = P \sin\,[Q\,(t/s)]$$

which describes its motion.

J78/I/1

48 A certain mass, suspended from a spring, performs vertical oscillations of period T when on Earth. If the system were transferred to the Moon, where the acceleration of free fall is one-sixth of that on Earth, what would be the period?

J80/I/1

49 A particle rests on a horizontal platform which is moving vertically in simple harmonic motion with an amplitude of 50 mm. Above a certain frequency, the particle ceases to remain in contact with the platform throughout the motion.

(a) Find the lowest frequency at which this occurs.

(b) At this minimum frequency, at what point in the motion does contact cease?

[Take the acceleration of free fall, g, as 10 m s^{-2}.]

J81/I/4

50

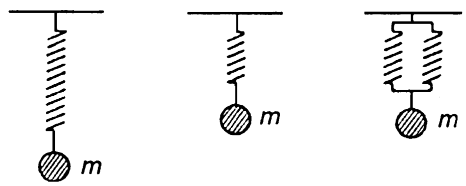


Fig. 10.1 **Fig. 10.2** **Fig. 10.3**

The period of vertical oscillation of a mass m at the end of a light helical spring of force constant k (Fig. 10.1) is T_1. (The force constant is the force per unit extension of the spring.) Write down the relation between T_1, m and k. If the spring were cut into two pieces of equal length and one portion were used to support the same mass (Fig. 10.2), what would be the period T_2? If both portions of the spring were used in parallel (Fig. 10.3), what would be the period T_3? (Give your answers for T_2 and T_3 in terms of T_1.)

J82/I/3

51 The motion of a piston in a certain car engine is approximately simple harmonic with amplitude 40 mm. The frequency of oscillation is 120 Hz. Find *(a)* the maximum acceleration, *(b)* the maximum speed, of the piston.

J83/I/3

52 A body of mass 0.15 kg moves with simple harmonic motion in a straight line. The relation between the force F acting on the body and its displacement x, over a complete oscillation, is shown in Fig. 11. Find *(a)* the amplitude of the motion, *(b)* its period, *(c)* the maximum speed of the body.

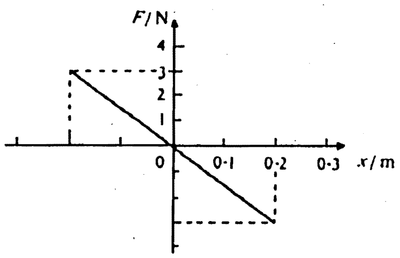


Fig. 11

N83/I/3

53 A mass suspended from one end of a helical spring undergoes vertical simple harmonic motion with an amplitude of 2.0 cm. If three complete oscillations are made in 4.0 s, what is the acceleration of the mass at *(a)* the equilibrium position, *(b)* the position of maximum displacement? N85/III/1

54 A horizontal plate is vibrating vertically with simple harmonic motion at a frequency of 20 Hz. What is the maximum amplitude of vibration so that fine sand on the plate always remains in contact with it? J86/III/1

55 A mass on the end of a light helical spring is given a vertical displacement of 3.0 cm from its rest position and then released. If the subsequent motion is simple harmonic with a period of 2.0 s, through what distance will be bob move in

(a) the first 1.0 s,

(b) the first 0.75 s? [6]

J88/III/1

56 A body undergoes simple harmonic motion such that its displacement x from the mean position is given by

$$x = x_0 \sin \omega t,$$

Sketch graphs, one in each case, to show the variation with time t of (i) the velocity of the body and (ii) its kinetic energy. [5]

N88/III/1

57 In order to check the shutter speed of a camera, a photograph is taken of a simple pendulum of period 2.0 s and amplitude 0.030 m. Examination of the photograph shows that the shutter opened as the pendulum bob passed the equilibrium position and closed after it had moved 0.018 m. Calculate the time for which the shutter was open. [4]

J89/III/3

58 The displacement x of a particle at time t in a sinusoidal wave is given by the expression

$$x = x_0 \sin \omega t.$$

Write down an expression, in terms of x_0, ω and t, for the displacement in a wave of half the intensity and double the frequency. [4] N89/III/1

59 *(a)* Explain the term simple harmonic motion. [2]

(b) (i) State the defining equation for simple harmonic motion. [1]

(ii) Write down a solution to the equation giving the displacement x in terms of the amplitude of oscillation x_0, the angular frequency ω and the time t. [1]

(c) Given that the velocity v of a body of mass m undergoing simple harmonic motion is given by

$$v = \pm\omega\sqrt{(x_0^2 - x^2)}$$

find the kinetic energy of the body in terms of its displacement x, and ω and x_0. [1]

(d) Sketch graphs, using the same horizontal axis, to show how the velocity and the kinetic energy vary with the displacement of a body undergoing simple harmonic motion. [3]

J90/II/3

60 *(a)* Calculate the gain in potential energy when a mass of 150 g is raised through 1.0 mm. [2]

(b) A simple pendulum consists of a light inextensible string to which is attached a bob mass 150 g. The variation of V_p, the potential energy, with x, the horizontal displacement of the bob, is shown in Fig. 12.

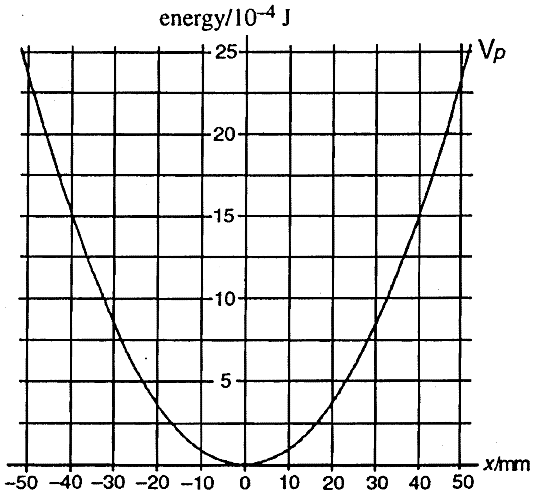


Fig. 12

In order to set the pendulum into oscillation, the bob is displaced sideways (keeping the string taut) until its centre of mass is raised vertically through 1.0 mm and then released. Using the axes of Fig. 12, sketch labelled graphs to show the variation, as the pendulum oscillates, of x with

(i) the total energy,

(ii) the kinetic energy. [4]

(c) By reference to Fig. 12, or otherwise, write down the amplitude of oscillation of the pendulum. [2]

N90/II/3

61 A light platform is supported by two identical springs, each having spring constant 20 N m^{-1}, as shown in Fig. 13.

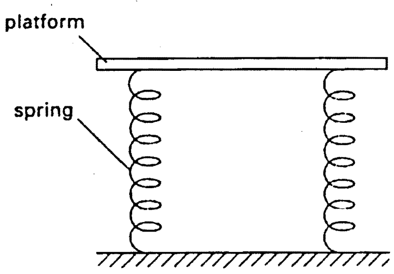


Fig. 13

(a) Calculate the weight which must be placed on the centre of the platform in order to produce a displacement of 3.0 cm. [2]

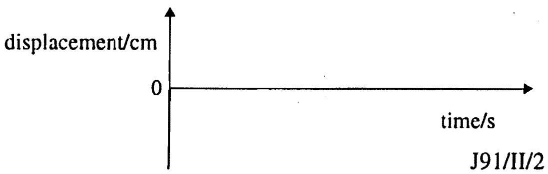
(b) The weight remains on the platform and the platform is depressed a further 1.0 cm and then released.

(i) What is the frequency of oscillation of the platform?

(ii) On the axes below, sketch a graph to show the variation with time of the displacement of the platform during the first second. Label your axes with appropriate numbers. [4]

(c) (i) Mark on your sketch the times at which the magnitude of the acceleration of the platform is maximum.

(ii) Calculate this maximum acceleration. [4]



J91/II/2

62 A mass of 0.100 kg oscillates with simple harmonic motion of amplitude 0.0030 m and period 0.020 s.

(a) Find the frequency of the oscillation. [1]

(b) Find ω, the angular frequency of the oscillation. [1]

(c) Write down the equation representing the variation with time, t, of the displacement, x, for this oscillation. [2]

(d) A graph of velocity against displacement for this oscillation is shown in Fig. 14.

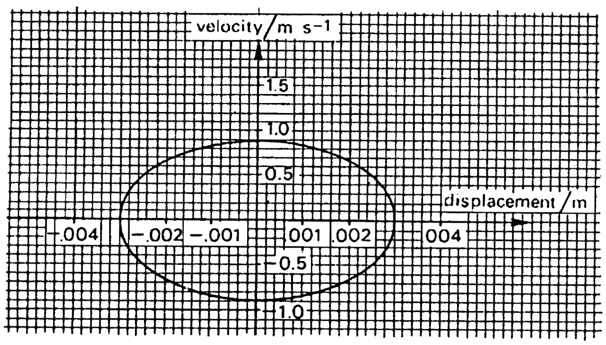


Fig. 14

(i) Read from the graph the maximum value of the velocity.

(ii) Explain why there are two values of velocity for zero displacement.

(iii) Explain why there are two values of displacement for zero velocity. [3]

(e) (i) Calculate the maximum kinetic energy of the mass.

(ii) Sketch on Fig. 15 a graph of the kinetic energy of the mass against displacement.

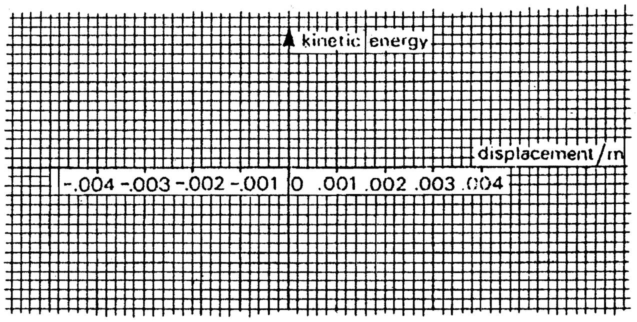


Fig. 15 [3]

J92/II/2

63 *(a)* State the equation defining simple harmonic motion. [1]

(b) The graph, Fig. 16, shows how the acceleration of an object undergoing simple harmonic motion varies with time.

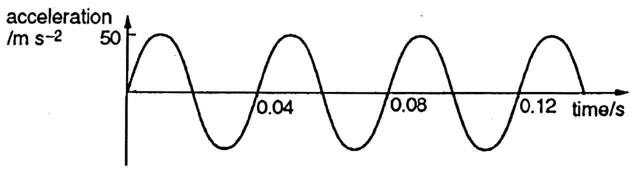


Fig. 16

Deduce, from the numerical values given on the graph, the values for this simple harmonic motion of

(i) the period,

(ii) the frequency,

(iii) the angular frequency ω,

(iv) the amplitude x_0 of the oscillation. [6]

(c) Sketch on Fig. 17 a graph which shows how the displacement varies with time.

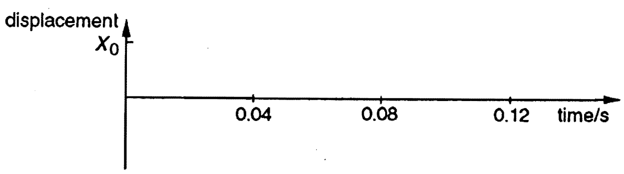


Fig. 17 [2]

N93/II/3

64 The pendulum bob in a particular clock oscillates so that its displacement from a fixed point is as shown in Fig. 18.

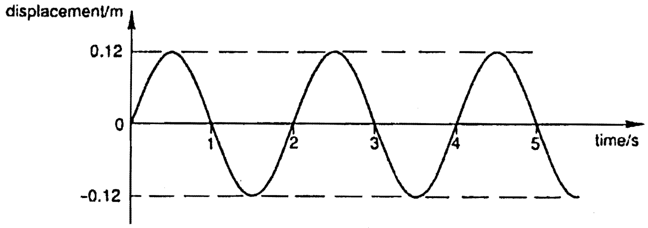


Fig. 18

By taking the necessary readings from the graph, determine for these oscillations,

(a) the amplitude, [1]

(b) the period, [1]

(c) the frequency, [1]

(d) the angular frequency, [1]

(e) the acceleration

(i) when the displacement is zero,

(ii) when the displacement is at its maximum, [3]

(f) the maximum velocity of the pendulum bob.
[Hint: $v = \pm\omega\sqrt{(x_0^2 - x^2)}$] [2]

N95/II/2

65 *(a)* An object undergoing simple harmonic motion has displacement from its equilibrium position. The displacement varies with time in the way shown in Fig. 19 *(a)*.

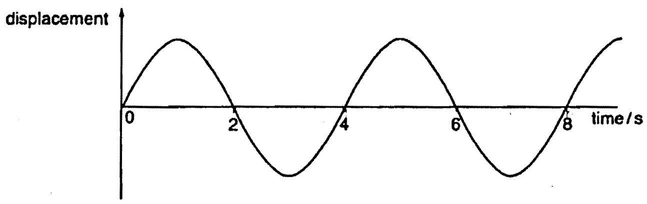


Fig. 19*(a)*

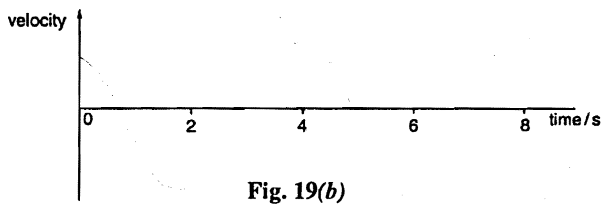


Fig. 19*(b)*

On Fig. 19 *(b)*, sketch the variation with time of the velocity of the object. [2]

(b) For the object in *(a)*, find

(i) the frequency f of the oscillation,

(ii) the angular frequency ω of the oscillation,

(iii) the phase difference between the displacement and the velocity. [4]

(c) The mass of an astronaut in an orbiting space station cannot be measured by using a normal balance. However, the mass can be measured by monitoring the oscillations of the astronaut when seated in a chair supported by a spring. The period of the oscillation T is given by the expression

$$T = 2\pi\sqrt{\frac{M}{k}},$$

where M is the total mass of the chair and the astronaut, and k is the spring constant.

For a particular chair, of mass 6.3 kg, the spring to which it is attached has a spring constant of 1540 N m^{-1}.

(i) Calculate the period of oscillation when an astronaut of mass 73.2 kg sits in the chair.

(ii) Calculate the percentage change in the period of oscillation after the mass of the astronaut increased by 0.5 kg during a meal. [5] J96/II/2

66 A vertical peg is fixed to the rim of a horizontal turntable of radius r, rotating with a constant angular speed ω, as shown in Fig. 20.

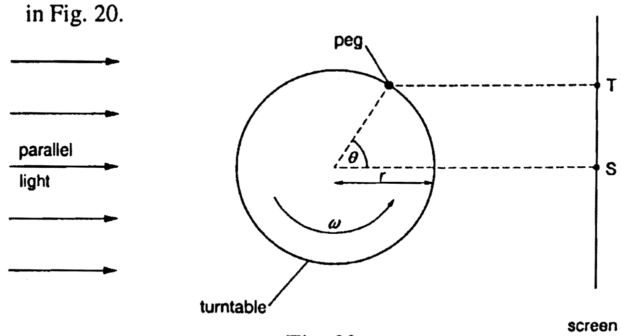


Fig. 20

Parallel light is incident on the turntable so that the shadow of the peg is observed on a screen which is normal to the incident light. At time $t = 0$, $\theta = 0$ and the shadow of the peg is seen at S.

At some later time t, the shadow is seen at T.

(a) (i) Write down an expression for θ in terms of ω and t.

(ii) Derive an expression for the distance ST in terms of r, ω and t. [2]

(b) By reference to your answer to *(a)*(ii), describe the motion executed by the shadow on the screen. [1]

(c) The turntable has a radius r of 20 cm and an angular speed ω of 3.5 rad s^{-1}. Calculate, for the motion of the shadow on the screen,

(i) the amplitude,

(ii) the period,

(iii) the speed of the shadow as it passes through S,

(iv) the magnitude of the acceleration of the shadow when the shadow is instantaneously at rest. [8]

N96/II/2

Long Questions

67 Discuss what is meant by a *simple harmonic motion* and define its *frequency, amplitude* and *angular frequency.* Deduce a formula for the period of small vertical oscillations of a mass attached to one end of a light, helical spring the other end of which is fixed.

Discuss in what way, if any, the motion of the system might differ if the fixed end of the spring were attached to a space ship in free orbit, i.e. engines switched off.

Define clearly all quantities you use. N76/III/1

68 What is meant by *simple harmonic motion*? Illustrate your answer with graphs and explain how these are related to uniform motion in a circle.

A dock has a tidal entrance at which the water is 10 m deep at 12 noon, when the tide is at its lowest. The water is 30 m deep when the tide is at its highest, which follows next at 6.15 p.m. A tanker, needing a depth of 15 m, requires to enter the dock as soon as possible that afternoon. Calculate the earliest time it could just clear the dock entrance.

State what you have assumed about tidal motion, and discuss critically what other factors might affect the earliest possible entry time in practice.

J77/III/1

69 Define *simple harmonic motion.*

Give an account, quoting any appropriate formulae, of the energy transformations that take place during such a motion. [6]

A clock has a 'balance' wheel that performs s.h.m. of period 0.5 s with a maximum angular displacement of π rad. Calculate the maximum angular velocity of the wheel. Explain your calculation carefully. [6]

N77/III/1 (part)

70 Define *(a)* the *frequency, (b)* the *angular frequency,* of an oscillation.

Simple harmonic motion may be regarded as the motion of the projection of a particle undergoing uniform circular motion on to a diameter. Draw a diagram to illustrate this statement and deduce an expression for the displacement of a particle undergoing simple harmonic motion in terms of the angular frequency and the amplitude of the motion. Deduce also an expression for the maximum kinetic energy of such a particle in terms of the frequency of the motion and its amplitude.

J82/III/1 (part)

71 Explain the meaning of *(a) displacement, (b) acceleration.* Define *simple harmonic motion* in terms of these quantities.

J84/III/1 (part)

72 Define *(a) displacement, (b) amplitude, (c) angular frequency,* of a simple harmonic motion and give an expression relating them, explaining all symbols used.

A student is under the impression that ω, the angular frequency of oscillation of a simple pendulum is dependent solely upon the length l of the pendulum and the mass m of its bob. Show, by dimensional analysis, that this cannot be correct. Derive from first principles the correct equation,

$$\omega^2 = g/l,$$

where g is the acceleration of free fall.

A small spherical mass is hung from the end of an elastic string of natural length 40·0 cm and when the pendulum so formed is set swinging with small amplitude, 20 oscillations are completed in 26·0 s. The bob is then replaced by one of the same size but of a different mass and the new time for 20 oscillations is 26·4 s. Account for this change and calculate the ratio of the masses. J85/III/9

73 *(a)* Explain the terms

(i) *potential energy,*

(ii) *kinetic energy.*

(b) A body of mass m oscillates freely in a straight line such that at time t its distance x from a fixed point is given by

$$x = a \sin \omega t$$

where a and ω are constants. Derive an expression for the potential energy V as a function of t. Hence, or otherwise, deduce expressions for (i) the kinetic energy T as a function of t, (ii) the total energy associated with the motion.

Sketch graphs, on the same axis for t, to show the variations of V and T. N86/III/9 (part)

74 What do you understand by (i) *simple harmonic motion,* (ii) *the amplitude* of such a motion?

When the mass M on the spring is 0.040 kg, the vertical displacement y of the mass varies with time t according to the relation

$$y = a \cos \omega t$$

where $a = 0.010$ m and $\omega = 20$ rad s^{-1}. What are

(a) the amplitude of the variation,

(b) the period T of the vibration?

Draw a sketch graph of vertical displacement against time to illustrate the motion and determine

(c) the equilibrium extension e,

(d) the force constant of the spring (the force per unit extension). J87/III/8 (part)

75 *(b)* The acceleration a of a particle undergoing simple harmonic motion is given by the expression

$$a = -bx,$$

where x is the displacement and b is a positive constant. How is b related to the period of the oscillation? [2]

(c) A vertical rod is fixed near the rim of a horizontal turntable which is rotating at exactly 33 revolutions per minute. A horizontal beam of light casts a shadow of the rod on to a screen in front of which is suspended a simple pendulum as shown in Fig. 21.

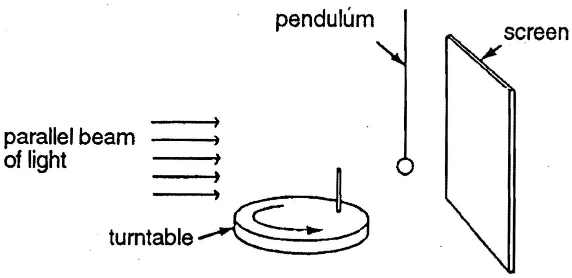


Fig. 21

If the shadows of the rod and the pendulum bob move in phase on the screen, what must be the effective length of the pendulum? [5]

(d) The speed of the turntable in *(c)* suddenly increases to $33\frac{1}{3}$ revolutions per minute.

(i) Briefly describe what will be observed subsequently on the screen.

(ii) Calculate the number of oscillations made by the pendulum before the two shadows are next in phase.

(iii) How long does this take? [7]

N89/III/9 (part)

76 *(a)* A long bar magnet hangs from one end of a spring, as shown in Fig. 22.

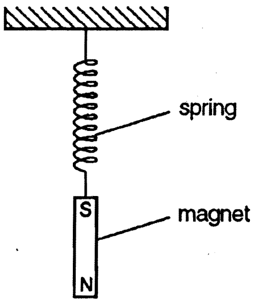


Fig. 22

The magnet is displaced vertically downwards and then released. The subsequent vertical displacement x is found to vary with time t as shown in Fig. 23.

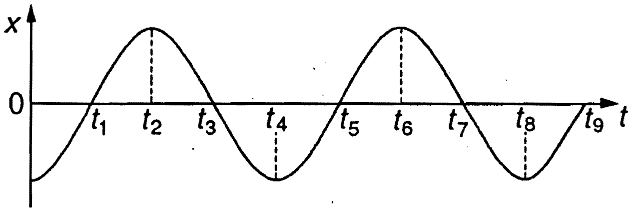


Fig. 23

(i) State **two** times, apart from $t = 0$, at which the magnet is stationary.

(ii) State **two** times at which the magnet is moving vertically upwards with maximum speed.

(iii) State **two** times at which the magnet is moving vertically downwards with maximum speed. [3]

J94/III/3 (part)

77 *(a)* In the preparation of tide tables for coastal resorts and harbours, use is made of a graph of depth of water against time at a particular place. One such graph is shown in Fig. 24.

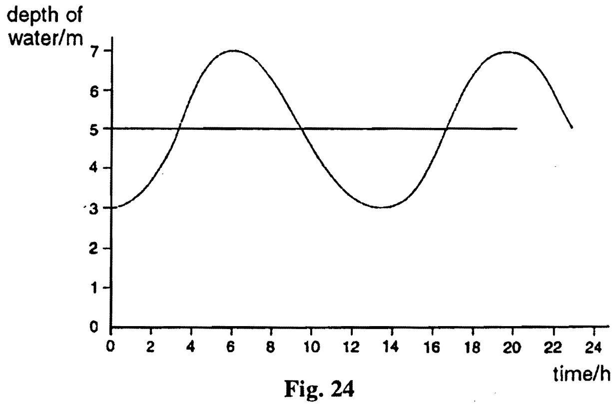


Fig. 24

(i) Describe the steps you would take to obtain such a graph practically for a particular harbour. [5]

(ii) What is the period of this oscillation? [1]

(iii) What is the amplitude of this oscillation? [1]

(b) In some harbours, the rise and fall of the water level is simple harmonic. What does *simple harmonic* mean? [3]

(c) In one harbour, the equation for the depth h of water is

$$h = 5.0 + 3.0 \sin \frac{2\pi r}{45600},$$

where h is given in metres and t is the time in **seconds**. (The angle $2\pi t/45600$ is in radians.) For this harbour, calculate

(i) the maximum depth of water, [1]

(ii) the minimum depth of water, [1]

(iii) the time interval between high- and low-water, [2]

(iv) two values of t at which the water is 5.0 m deep, [2]

(v) the length of time for each tide during which the depth of water is more than 7.0 m. [4]

J95/III/3

78 *(a)* (i) Explain what is meant by the *frequency* of vibration of an object.

(ii) Distinguish between the *displacement* of a vibrating object and the *amplitude* of vibration. [3]

(b) Some sand is placed on a flat horizontal plate and the plate is made to oscillate with simple harmonic motion in a vertical direction, as illustrated in Fig. 25.

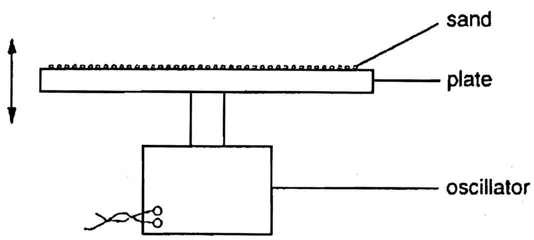


Fig. 25

The plate oscillates with a frequency of 13 Hz.

(i) Sketch a graph to show the variation with displacement x of the acceleration a of the plate.

(ii) The acceleration a is given by the expression

$$a = -\omega^2 x,$$

where ω is the angular frequency. Calculate

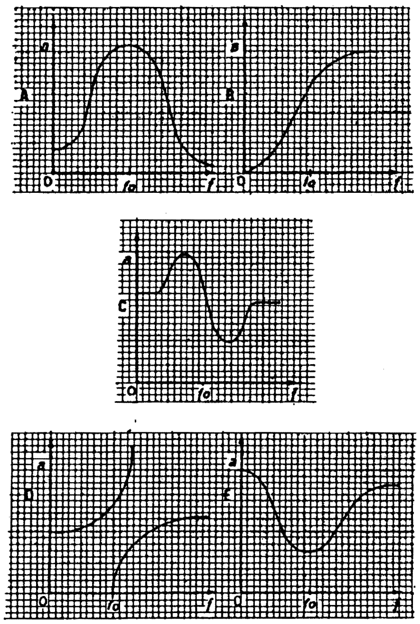
1. the angular frequency ω,
2. the amplitude of oscillation of the plate such that the maximum acceleration is numerically equal to the acceleration of free fall. [8]

(c) Suggest, with a reason, what happens to the sand on the plate in *(b)* when the amplitude of oscillation of the plate exceeds the value calculated in *(b)*(ii)2. [3]

J99/III/4 (part)

Oscillations & Resonance

79 A periodic impulse of frequency f is applied to a vibrating system of natural frequency f_0. Which one of the following graphs best represents the way in which the amplitude of vibration, a of the system varies with the frequency f?



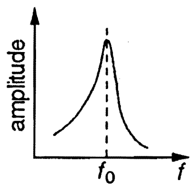
J77/II/10; N77/II/11

80 Because of air resistance, the amplitude of oscillation of a simple pendulum decays exponentially with time. How does the total energy of the pendulum vary with time?

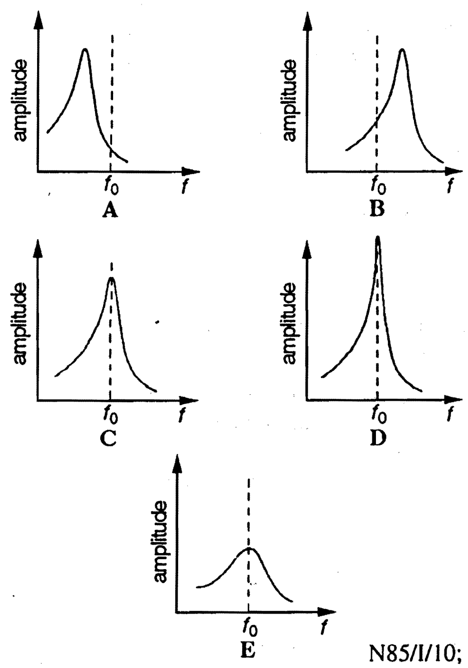
A It decays at a steady rate.
B It decays exponentially.
C It remains constant.
D It oscillates about zero with the same frequency as the pendulum.
E It oscillates about zero with twice the frequency of the pendulum.

N81/II/9

81 A pendulum is constructed from a fixed length of light thread and a spherical, low density, polystyrene bob. It is forced to oscillate at different frequencies f in air, and the response is shown in the graph below.

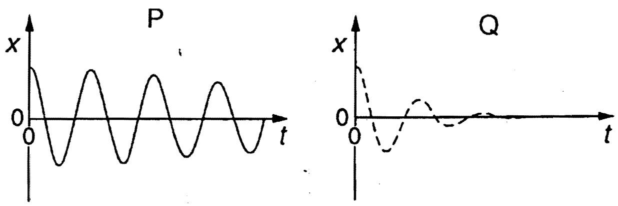


Which one of the following graphs best represents the results if the experiment were to be repeated in a vaccum?



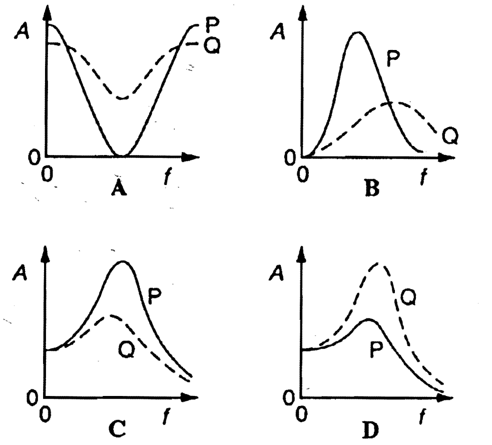
N85/I/10; N90/I/12

82 Two objects P and Q are given the same initial displacement and are then released. The graphs show the variation with time t of their displacements x.



P and Q are then subjected to driving forces of the same constant amplitude and of variable frequency f.

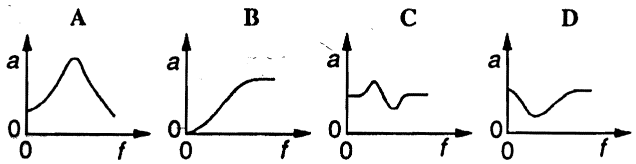
Which graph represents the variation with f of the amplitudes A of P and of Q?



J86/I/8; N93/I/7; J99/I/9

83 A pendulum is driven by a sinusoidal driving force of frequency f.

Which graph best shows how the amplitude a of the motion of the pendulum varies with f?

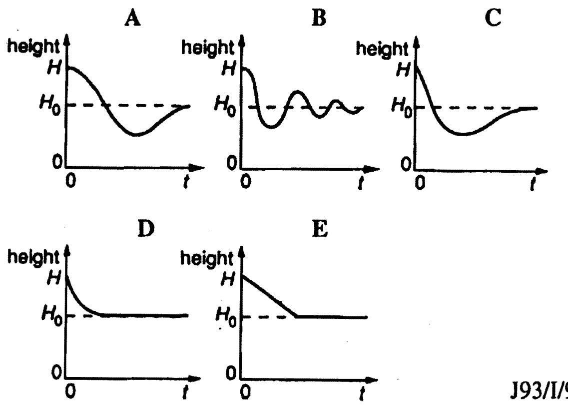


N89/I/9; N2000/I/9

84 It is important that a car suspension system should be critically damped.

The equilibrium height above the ground of the bodywork of such a car is H_0. The body of the car is raised to a greater height H and released at time $t = 0$. Assume that the car tyres remain in contact with the ground throughout and there is critical damping.

Which graph shows how the height of the car body above the ground varies with time?



J93/I/9

85 A lump of plasticine is dropped from a height on to the pan of a compression spring balance and remains attached to the pan. Give a qualitative explanation of the subsequent readings of the balance.

N76/I/2

86 The key on a piano corresponding to the note of frequency 440 Hz is depressed very gently so that the spring is free to vibrate. When the key corresponding to the note of frequency 220 Hz is struck, it is found that the 440 Hz string emits a note of frequency 440 Hz.

(a) Give a brief explanation of this observation.

(b) If the 110 Hz string had also been free to vibrate, what frequencies (if any) would it have emitted when the 220 Hz key was struck?

J78/I/5

87 Sketch a set of graphs on the same axes to show how the amplitude of forced oscillations of a resonant system varies with the driving frequency for

(i) very light damping,

(ii) moderate damping,

(iii) heavy damping.

J85/III/3

88 A block of wood of mass m floats in still water, as shown in Fig. 26.

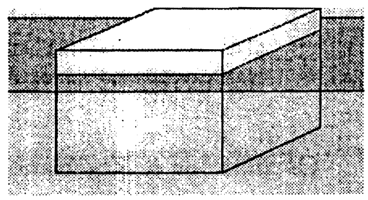


Fig. 26

When the block is pushed down into the water, without totally submerging it, and is then released, it bobs up and down in the water with a frequency f given by the expression

$$f = \frac{1}{2\pi}\sqrt{\frac{28}{m}}$$

where f is measured in Hz and m in kg.

Surface water waves of speed 0.90 m s^{-1} and wavelength 0.30 m are then incident on the block. These cause resonance in the up-and-down motion of the block.

(a) Explain what is meant by the term *resonance*. [2]

(b) Calculate

(i) the frequency of the water waves,

(ii) the mass of the block. [3]

(c) Describe and explain what happens to the amplitude of the vertical oscillations of the block after the following changes are made independently:

(i) water waves of larger amplitude are incident on the block,

(ii) the distance between the wave crests increases,

(iii) the block has absorbed some water. [6]

N94/II/2

89 Fig. 27 illustrates a mass which can be made to vibrate vertically between two springs.

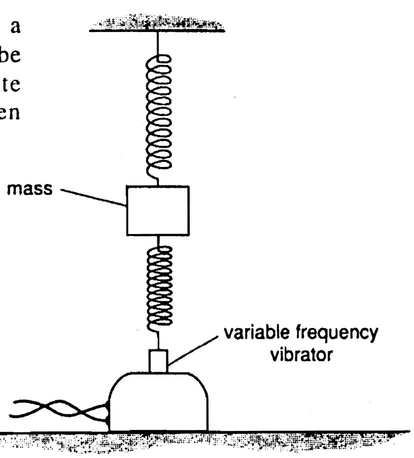


Fig. 27

The vibrator itself has constant amplitude. As the frequency is varied, the amplitude of vibration of the mass is seen to change as shown in Fig. 28.

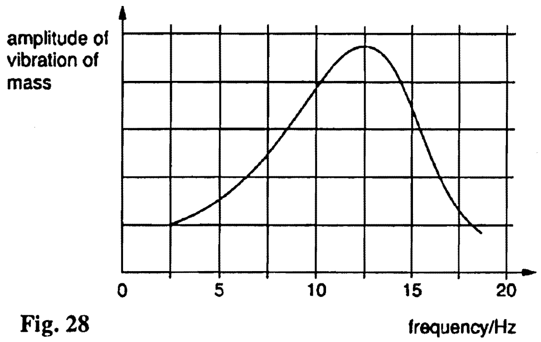


Fig. 28

(a) Name the phenomenon which is illustrated in Fig. 28. [1]

(b) For the mass vibrating at maximum amplitude, calculate

(i) the angular frequency,

angular frequency =...

(ii) the period.

period =.. s [4]

(c) A light piece of card is fixed to the mass with its plane horizontal. On Fig. 28, draw a line to show the variation with frequency of the amplitude of vibration of the mass. [2]

(d) State one situation in which the phenomenon illustrated in Fig. 28 is used to advantage. [1]

J97/II/2

90 A light spring hangs vertically from a fixed support and a mass is attached to its free end as illustrated in Fig. 29.

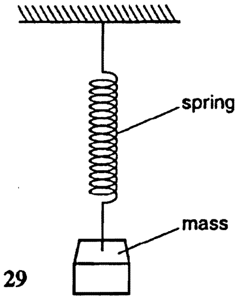


Fig. 29

The mass is displaced vertically and then released. The variation with time t of the displacement d of the mass from its equilibrium position is shown in Fig 30.

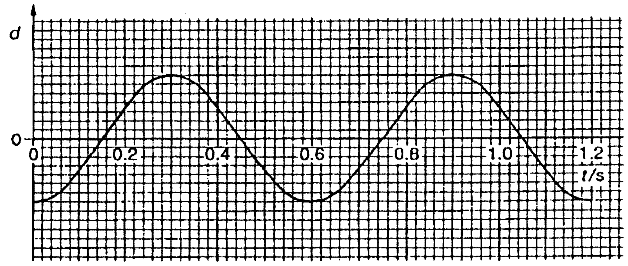


Fig. 30

(a) Use Fig. 30 to determine, for the oscillation of the mass,

(i) the period, (ii) the angular frequency. [3]

(b) A separate similar mass-spring system is set into oscillation and the variation with time t of the displacement d is shown in Fig. 31. The origin of time in Fig. 31 is the same as that in Fig. 30.

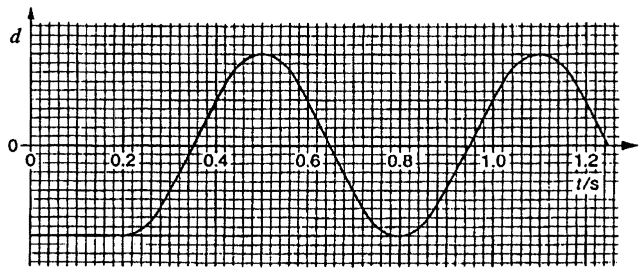


Fig. 31

Use Fig. 30 and Fig. 31 to determine

(i) the time interval between the start of the oscillations of the two masses,

(ii) the phase angle, in radians, between the two oscillations. [3]

(c) The mass-spring system of Fig. 29 is used to demonstrate the effects of damping on the oscillations of the mass.

(i) Explain what is meant by *damping*.

(ii) Suggest how

1. light damping of the oscillations may be achieved,

2. the degree of damping may be increased. [4]

N2000/II/3

Long Questions

91 Define *frequency of oscillation.* Explain what is meant by *damped oscillation,* and *forced oscillation.*

Describe an experiment to demonstrate how the sharpness of resonance in an oscillating system depends on the degree of damping. J76/III/1 (part)

92 Outline one experiment that suggests that light waves are transverse.

A loaded test tube floats vertically at one end of a ripple tank. The frequency of a constant-amplitude wave generator at the other end is varied until the amplitude of vertical oscillation of the test tube is a maximum.

In terms of the quantities listed below, derive a formula for

(a) the natural frequency of a loaded test tube in a liquid,

(b) the wavelength of the ripples when the amplitude is a maximum. Calculate this wavelength from the data.

Diameter of test tube, $D = 1.2 \times 10^{-2}$ m ;
Mass of test tube, $m = 2.5 \times 10^{-2}$ kg ;
Density of water, $\rho = 1.0 \times 10^{3}$ kg m^{-3} ;
Speed of ripples at resonance, $v = 7.5 \times 10^{-2}$ m s^{-1} ;
Acceleration of free fall, $g = 9.8$ m s^{-2}.

In practice, the calculated and measured values of this wavelength differ significantly. Discuss why this may arise.

J78/III/1

93 Define *simple harmonic motion.* How would you determine whether such a motion remains simple harmonic at large amplitude?

A motor car is driven at steady speed over a rough road on which the surface height varies sinusoidally. The shock absorber mechanism which normally damps vertical oscillation is not working. At a certain critical speed, the amplitude of vertical oscillation of the car becomes very large. Explain why this happens.

In term of the quantities listed below, derive a formula for

(a) the natural frequency of vertical oscillation of the car,

(b) the critical speed when the amplitude of vertical oscillation is a maximum. Calculate this speed from the data.

Mass of car and passenger, $M = 2.0 \times 10^{3}$ kg ;
Vertical rise of car when passengers get out, $S = 1.0 \times 10^{-1}$ m ;
Mass of passengers, $m = 5.0 \times 10^{2}$ kg ;
Wavelength of road surface corrugations, $\lambda = 20$ m ;
Acceleration of free fall, $g = 9.8$ m s^{-2}.

Discuss the behaviour of the car when the shock absorber mechanism is working correctly, giving appropriate sketch graphs.

N78/III/1

94 *(a)* What do you understand by

(i) the amplitude of a simple harmonic motion,

(ii) damped simple harmonic motion,

(iii) forced oscillations,

(iv) resonance?

(b) Describe an experiment

(i) to measure the natural frequency of a lightly damped mechanical system capable of s.h.m.,

(ii) to show that this system resonates at a frequency close to its natural frequency.

(c) Sketch a set of graphs, using the same axes, to show how the amplitude of forced oscillation varies with driving frequency for

(i) very light damping,

(ii) moderate damping,

(iii) heavy damping.

N81/III/1

95 *(c)* A small poor quality microphone has a *natural frequency of vibration* of 13 kHz. A sound wave of constant pressure *amplitude* but variable frequency is incident upon the microphone.

(i) What is meant by the terms printed in *italics*?

(ii) The microphone responds to pressure fluctuations in the sound wave. If the output of the microphone is dependent solely upon its amplitude of vibration explain, with the aid of a sketch graph, how the output will vary as the frequency of the sound wave is increased from 10 kHz to 16 kHz.

N86/III/9 (part)

96 Define *simple harmonic motion.* How would you investigate experimentally whether the motion of a pendulum remains simple harmonic as the amplitude of vibration is increased? [6]

The suspension of a car may be considered to be a spring under compression combined with a shock absorber which damps the vertical oscillations of the car. Draw sketch graphs, one in each case, to illustrate how the vertical height of the car above the road will vary with time after the car has just passed over a bump if the shock absorber is

(a) not functioning, i.e. slides without resistance,

(b) operating normally. [4]

When the driver, of mass 80 kg, steps into the car, of mass 920 kg, the vertical height of the car above the road decreases by 2.0 cm. If the car is driven over a series of equally spaced bumps, the amplitude of vibration becomes much larger at one particular speed. Explain why this occurs and calculate the separation of the bumps if it occurs at a speed of 15 m s^{-1}. [7]

(The frequency of vibration of a loaded spring is $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where m is the mass on the spring and k is the force required to produce unit extension of the spring.)

N87/III/8

97 *(a)* (i) Give an account of the energy transformations which occur during one complete oscillation of an undamped simple pendulum. Illustrate you answer with sketch graphs wherever appropriate. [6]

(ii) Explain why a lightly damped oscillating system experiences a progressive decrease in its amplitude. [2] N89/III/9 (part)

98 *(c)*

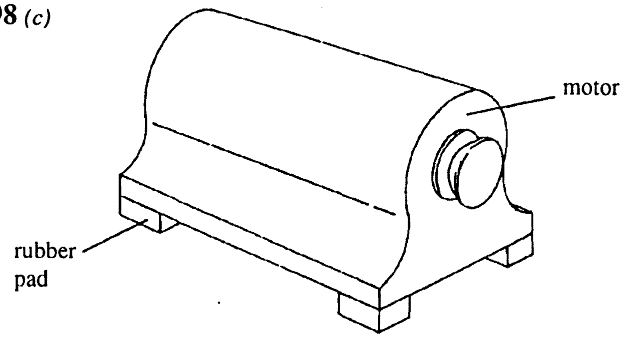


Fig. 32

In order to try to reduce vibration from an electric motor, it is isolated from the floor by four identical rubber pads as shown in Fig. 32. One pad is placed at each corner.

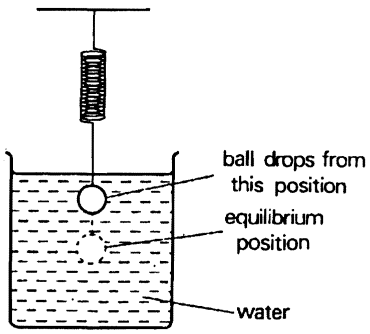
With the motor switched off, the pads are compressed and then released. The subsequent vertical oscillations have frequency 10 Hz.

(i) Sketch a labelled graph to show how the amplitude of vertical oscillations will vary with frequency of rotation of the motor when its speed of rotation is slowly increased from a low value to 25 Hz.

(ii) Describe and explain how your graph in *(c)* (i) will change, if at all, when the motor is supported with a further four identical rubber pads which are placed at each corner, so that the thickness of each composite pad is doubled. [8]

N91/III/3 (part)

99 *(a)* Fig. 33 illustrates two vertical oscillating motions. In Fig. 33 (i) a ball, suspended from a spring, is immersed in water. In Fig. 33 (ii) a ball bounces on the ground.



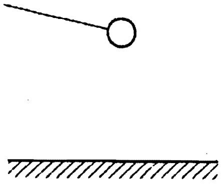


Fig. 33 (i) **Fig. 33 (ii)**

Sketch labelled graphs to show possible variations of velocity with time for these motions. [8]

(b) Describe how, in the motion of the ball on the spring, the direction of the acceleration varies with

(i) the direction of the velocity,

(ii) the direction of the resultant force. [4]

(c) The ball on the spring is said to be undergoing damped, simple harmonic motion.

(i) Describe what is meant by *simple harmonic motion.* [2]

(ii) State two changes which may be made to the apparatus in order to alter the degree of damping. [2]

(iii) What effects does the degree of damping have on the movement of the ball? [4]

N92/III/2

100 *(a)* A load of mass m is suspended from the free end of a helical spring of spring constant k, as shown in Fig. 34.

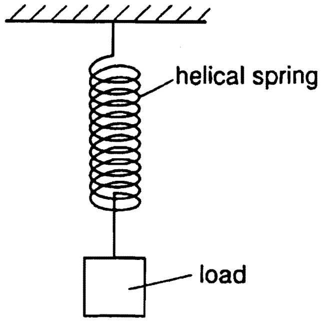


Fig. 34

The load is displaced vertically and then released. The load oscillates with frequency f given by the expression

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} .$$

(i) Explain what is meant by

1. the *spring constant* of the spring,
2. the *oscillation* of the load.

(ii) Motion sensors are used to monitor the movement of the load, and the variation with time t of the position of the load is as shown in Fig. 35.

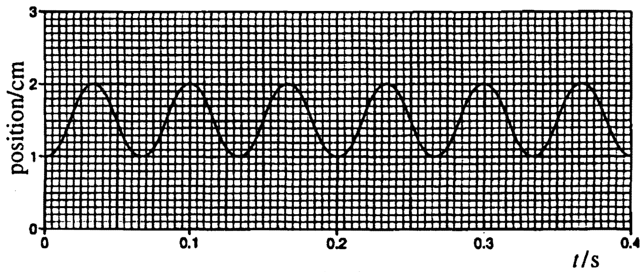


Fig. 35

Use Fig. 35 to

1. suggest, with a reason, whether the motion is damped or undamped,
2. calculate the spring constant k, given that the mass of the load is 90 g. [8]

(c) In outer space where the gravitational field strength is zero, springs are used to compare the mass of two objects. Suggest how this is achieved. [4]

N97/III/3 (part)

101(a) Describe an example of a free oscillation. Explain why in practice a free oscillation cannot have a constant amplitude. [2]

(b) An object undergoing a forced oscillation has displacement y, as shown in the graph of Fig. 36.

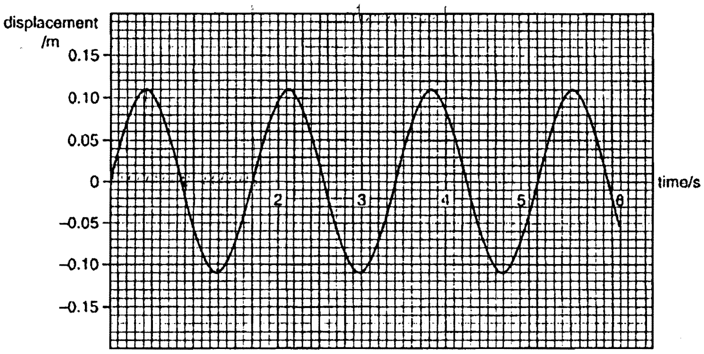


Fig. 36

Use the graph to determine, for this oscillation,

(i) its amplitude,

(ii) its period,

(iii) its frequency,

(iv) its angular frequency. [5]

(c) Use Fig. 36. to state, for each of the following, a time at which the oscillating object has

(i) maximum positive velocity,

(ii) maximum positive acceleration,

(iii) maximum negative acceleration,

(iv) maximum kinetic energy,

(v) maximum potential energy. [5]

(d) A driver of constant amplitude and variable frequency f causes forced oscillations of an object. The amplitude y_0 of the object's oscillations depends on f.

(i) Sketch a graph to show how y_0 varies with f over a wide range of frequencies which includes the natural frequency of the object.

(ii) Add to your sketch a second line which shows the effects of increased damping. Label this line D. [4]

(e) The phenomenon which you have illustrated in (d)(i) can cause considerable engineering problems. Explain one such problem and suggest ways in which it can be overcome. [4]

N98/III/2