## TOPIC 5 Forces

## Hooke's Law

1 A wire is stretched by a force F which causes an extension 1 . The energy stored in the wire is $1 / 2 \mathrm{~F}$ only if

A the extension of the wire is proportional to the force applied.
B the weight of the wire is negligible.
C the wire is not stretched beyond its elastic limit.
D the acceleration of free fall is constant.
E the cross-sectional area of the wire remains constant.
N76/II/40
2 The force $F$ required to extend a sample of rubber by a length $x$ was found to vary as shown in the diagram.


The energy stored in the rubber for an extension of 5 m was
A more than 200 J .
D $\quad 100 \mathrm{~J}$.
B $\quad 200 \mathrm{~J}$.
E less than 100 J .
C between 100 J and 200 J .

J78/II/36; J83/II/40; J89/I/30
3 A wire that obeys Hooke's law is of length $x_{1}$ when it is in equilibrium under a tension $T_{1}$; its length becomes $x_{2}$ when the tension is increased to $T_{2}$. What is the extra energy stored in the wire as a result of this process?
A $\quad 1 / 4\left(T_{2}+T_{1}\right)\left(x_{2}-x_{1}\right)$
D $\quad 1 / 2\left(T_{2}+T_{1}\right)\left(x_{2}+x_{1}\right)$
B $\quad 1 / 4\left(T_{2}+T_{1}\right)\left(x_{2}+x_{1}\right)$
E $\quad\left(T_{2}-T_{1}\right)\left(x_{2}-x_{1}\right)$
C $\quad 1 / 2\left(T_{2}+T_{1}\right)\left(x_{2}-x_{1}\right)$
J81/II/39; N87/I/30

4 A sample is placed in a tensile testing machine. It is extended by known amounts and the tension is measured.


What is the work done on the sample when it is given a total extension of 9 mm ?
A 0.31 J
B 0.36 J
C 0.43 J
D $\quad 0.72 \mathrm{~J}$
N83/LI/40; J99/I/23

5 A uniform strip of rubber, marked with equal divisions, is fixed at end P and pulled into tension by a force $F$ applied at end Q , as shown in the diagram below.


Which one of the following diagrams correctly shows the separation of the divisions when the elastic is extended to twice its original length?
A H |
в $\square$ I $\square$ |ll|
C $\square$



N86/I/30
6 A force of 10 N acting on a certain spring gives an extension of 40 mm . Two such springs are connected end to end and this double-length spring is extended by 40 mm .
Assuming that the springs conform to Hooke's law, what is the strain energy?

| A | 0.05 J |
| :--- | :--- |
| B | 0.10 J |
| C | 0.20 J |
| D | 0.40 J |
| E | 0.80 J |

J88/I/30
7 Some weight-lifters use a 'chest expander', consisting of a strong spring with a handle at each end, to exercise chest and arm muscles.
For one such chest expander, the relation between the force $F$ applied by the weight-lifter and the extension $x$ of the spring is shown in the graph.


Which graph best shows how $W$, the work done, depends on the extention $x$ ?

A


C


D


E


N93/I/21

8 The graph shows the relationship between load $f$ ank extension $x$ for a certain spring.


A load of 6.0 N is placed on the spring.
What additional strain energy will be stored in the spring if it is then extended a further 0.01 m ?
A 0.010 J
C $\quad 0.070 \mathrm{~J}$
B $\quad 0.060 \mathrm{~J}$
D $\quad 0.160 \mathrm{~J}$

J94/I/22
9 A spring obeying Hooke's law has an unstretched length of 50 mm and a spring constant of $400 \mathrm{~N} \mathrm{~m}^{-1}$.
What is the tension in the spring when its overall length is 70 mm ?
A $\quad 8.0 \mathrm{~N}$
C $\quad 160 \mathrm{~N}$
B $\quad 28 \mathrm{~N}$
D $\quad 400 \mathrm{~N}$

N94/I/23
10 A door is filled with a spring-operated latch as shown.


The latch is well-oiled so friction is negligible.

When the latch is pushed in, the spring becomes compressed but remains within its elastic limit.

The latch is then suddenly released.

Which graph best shows how the acceleration $a$ of the latch varies with the distance $x$ it moves before it is stopped?

A
B


C




N97/I/4

11 A catapult consists of two strands of rubber, each of unstretched length 0.20 m and each of which stretches by 0.1 m under a tension of 50 N . A stolc of mass 0.060 kg is projected vertically upwards from the catapult after each striand has been extended to a total length of 0.35 m . What is the energy stored in the stretched catapult? Find the maximum height attained by the stone. (Assume that the rubber obeys Hooke's law and that air resistance is negligible.)
[Acceleration of free fall, $\mathrm{g}=10 \mathrm{~m} \mathrm{~s}^{-2}$.]
J76/I/II
12 The scale of a certain spring balance reads from 0 to 100 N and is 0.15 m long. Find the strain energy of the spring when the balance reads 40 N . (Assume that the spring obeys Hooke's law.)

J86/II/2
13 The force constant $k$ of a spring is the constant of proportionality in the Hooke's law relation $T=\mathrm{ke}$ between tension $T$ and extension $e$.


Fig. 1
A spring $A$ of force constant $6 \mathrm{~N} \mathrm{~m}^{-1}$ is connecied in series with a spring $B$ of force constant $3 \mathrm{Nm}^{-1}$, as shown in Fig. 1 . One end of the combination is securely anchored and a force of 0.6 N is applied to the other end.
(a) By how much does each spring extend?
(b) What is the force constant of the combination? J87/II/7

## Long Questions

14 The following values of the extension $e$ of a wire were obtained for various applied forces $F$ :

| $e / \mathrm{mm}$ | 0 | 0.7 | 1.4 | 2.1 | 2.8 | 3.2 | 3.8 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $F / \mathrm{N}$ | 0 | 20 | 40 | 60 | 80 | 90 | 100 |

All the extensions were within the elastic limit. The original length of the wire was 2.20 m and its area of cross-section was $0.53 \mathrm{~mm}^{2}$.
(a) Make a labelled diagram of an experimental arrangement suitable for applying the force and measuring of the extension.
(b) Draw a graph of $e$ against $F$ and use it to find the work done in giving the wire an extension of 3.8 mm .

N82/l/18 (part)
15 The table below shows how the extension $e$ of a 10 m length of a certain nylon climbing rope depends on the applied force $F$.

| $e / \mathrm{m}$ | 0 | 1.9 | 2.8 | 3.4 | 3.8 | 4.1 | 4.3 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $F / \mathrm{kN}$ | 0 | 2.0 | 4.0 | 6.0 | 8.0 | 10.0 | 12.0 |

(a) Draw a graph of applied force against extension.

A climber of mass 70 kg , attached to a 10 m length of this rope, can withstand a force from the rope of no more than 6.5 kN without the risk of serious injury.
(b) Read off from your graph the extension which would be produced in the rope for a force of 6.5 kN .
(c) Use the graph to find the energy stored in the rope if it were stretched by this amount.
(d) If the upper end of the rope were securely anchored, through what vertical distance could the climber fall freely (before the rope started to stretch) without risk of injury from the force of the rope when his fall was arrested?

J85/II/13 (part)
16 A wire is stretched by a force $F$ which causes an extension $x$. Hooke's law relates $F$ to $x$, and may be expressed by the equation

$$
F=K x .
$$

(a) What are the dimensions of $K$ (the force constant of the wire?)
(c) If the wire were cut in half, what would be the force constant $K^{\prime}$ of one of the pieces in terms of the force constant $K$ of the original wire?
(d) Find an expression for the work done in stretching one of the cut pieces of wire in (c) from its original length to an extension $x_{1}$, assuming Hooke's law is obeyed. Give your answer in terms of $K^{\prime}$ and $x_{1}$. Explain how this work could be calculated if the wire did not obey Hooke's law for the whole of the extension $x_{1}$.

A thick rubber cord is fixed firmly at one end. A 5.0 kg mass is attached to the lower end, causing an equilibrium extension of 20 mm . The rubber obeys Hooke's law over this extension.
(i) Find the change in the gravitational potential energy of the mass as a result of the extension.
(ii) How much energy is stored in the rubber?
(iii) How do you reconcile your answers to (i) and (ii) with the principle of conservation of energy?

N86/II/13 (part)
17 (b) The spring constant $k$ of a spring may be determined by finding the extension of the spring and the load applied, using the apparatus shown in Fig. 2.

(c) A student obtained the following readings using the apparatus in Fig. 2.
reading on the rule for the lower $\quad=13.60 \pm 0.05 \mathrm{~cm}$ end of the unextended spring
reading on the rule for the lower $\quad=17.95 \pm 0.05 \mathrm{~cm}$
end of the extended spring
load

$$
=4.00 \pm 0.02 \mathrm{~N} .
$$

It may be assumed that the spring obeys Hooke's law.
(i) Estimate the percentage uncertainty in the determination of $k$.
(ii) Calculate $k$ and give it with its actual uncertainty to the appropriate number of significant figures. [7]
(d) What is the percentage uncertainty in the determination of the extension of the spring if the measurements made in (c) are obtained with a load of 2.00 N ?

N91/III/1 (part)
18 (b) (i) A force is required to cause an extension of a spring. Explain why this causes energy to be stored in the spring.
(ii) A spring of spring constant $k$ undergoes an elastic change resulting in an extension $x$. Deduce that $W$, its strain energy, is given by

$$
\begin{equation*}
W=1 / 2 k x^{2} . \tag{7}
\end{equation*}
$$

(c) A toy train, mass $m$, travels along a track at speed $v$ and is brought to rest by two spring buffers which are shown in Fig. 3.


Fig. 3
Each buffer has spring constant $k$.
(i) By considering the energy transfer, derive an expression to show how the maximum compression of the buffers varies with the initial speed of the train.
(ii) Calculate the maximum compression of the buffers for a train of mass $m=1.2 \mathrm{~kg}$ travelling with an initial speed $v=0.45 \mathrm{~m} \mathrm{~s}^{-1}$ when the spring constant $k$ of each buffer is $4.8 \times 10^{3} \mathrm{Nm}^{-1}$.
[5]
State and explain a reason why, in practice, spring buffers of this design are not used.
[2]

19 (b) A cylinder of length 1.20 m , area of cross-section $8.30 \times 10^{-5} \mathrm{~m}^{2}$, and mass 0.777 kg falls from rest so that its base drops from a height of 2.30 m above the ground, as shown in Fig. 4.


Fig. 4
(i) Calculate, ignoring air resistance,

1. the speed of the cylinder just before it hits the ground,
2. the kinetic energy of the cylinder just before it hits the ground.
(ii) On hitting the ground, the cylinder may be considered to behave perfectly elastically and to obey Hooke's law. Under these conditions it undergoes a maximum contraction of $1.60 \times 10^{-3} \mathrm{~m}$. Calculate
3. the maximum elastic potential energy stored in the cylinder,
4. the average force the cylinder exerts on the ground,
5. the maximum force it exerts on the ground,
(iii) If the collision with the ground is not perfectly elastic, suggest qualitatively, with reasons, how your answers to (b) (ii) 2 and 4 might be different.

J95/III/I (part)
$\mathbf{2 0}$ (d) A wire, made of a ductile material of density $7.6 \times 10^{3} \mathrm{~kg} \mathrm{~m}^{-3}$, has an area of cross section $2.5 \times 10^{-7} \mathrm{~m}^{2}$ and initial length 2.00 m . It is stretched by applying a tensile force to it. The force increases to 48 N , at which value the extension increases from 8.0 mm to 88 mm , as shown in Fig. 5. Once the extension has reached 88 mm the force is removed and the wire is then seen to have been permanently stretched by 80 mm .


Fig. 5

## Calculate

(i) the mass of the wire,
(iii) the work done on the wire

1. while the extension increases from zero to 8.0 mm ,
2. while the extension increases from 8.0 mm to 88 mm .
(iv) Discuss how the work done which you calculated in the two parts of (iii), is transformed to different forms of energy.
(v) The specific heat capacity of the material of the wire is $450 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$. Estimate the rise in the temperature of the wire when the extension increases from zero to 88 mm .

J97/III/5 (part)
21 A metal wire of unextended length $/$ and cross-sectional area $A$ is supported at one end from a fixed point and a mass of weight $W$ is attached to its free end, as shown in Fig. 6.


Fig. 6
The wire is found to extend by a length $e$ when the weight is attached.
(a) (i) Derive expressions, in terms of $!, A, W$ ande $e$ for

1. the spring constant $k$ of the wire,

J98/III/4 (part)

## Forces

22


Fig. 7

Two immiscible liquids $P$ and $Q$ of different densities are contained in a wide U-tube as shown in Fig. 7. The heights of the two liquids above the horizontal line $X X^{\prime}$ which cuts the boundary between the liquids are $H_{p}$ and $H_{Q}$ respectively.
Surface tension effects and compressibility of the liquids can be neglected.

The U-tube is transported to a planet where the acceleration of free fall is $2 / 3$ that on Earth, where the liquid does not evaporate and where the heights of liquid (measured relative to $X X^{\prime}$ ) and $h_{p}$ and $h_{Q}$ respectively. Which one of the following is correct?

A The liquid levels are unchanged, so that $h_{P}=H_{P} ; h_{Q}=H_{Q}$
B Both liquid levels rise so that $h_{P} / h_{Q}=H_{P} / H_{Q}$
C Both liquid levels rise so that $\left(h_{P}-h_{\varrho}\right)=\left(H_{P}-H_{Q}\right)$
D Liquid $P$ rises and liquid $Q$ falls so that $h_{P} / h_{Q}=3 H_{P} / 2 H_{Q}$
E Liquid $P$ falls and liquid $Q$ rises so that $h_{p} / h_{Q}=2 H_{p} / 3 H_{Q}$
J76/II/8
23 A small brass sphere is held just below the surface of glycerine in a deep tank. It is then released and its position is plotted as a function of time after release. Which of the following graphs best represents the relation between the distance $s$ travelled by the sphere and the time $t$ ?




N76/II/7
24 A steel ball is held a short distance, $h$, above the surface of a liquid in a deep vessel and then released. The terminal velocity of the ball in the liquid is independent of

A the distance $h$.
B the density of the liquid.
C the diameter of the sphere.
D the viscosity of the liquid.
E the temperature of the liquid.
N77/II/5
25 A solid steel sphere is held just below the surface of water and released. Its terminal velocity $v$ is measured. If the experiment were repeated in places with different values of $g$ (the acceleration of free fall), a graph of $v$ against $g$ would most closely resemble




N78/II/5
26 Which one of the following graphs best represents how the acceleration of a small steel ball varies with time as it falls from rest in a tall beaker full of glycerine?



N78/II/2
27 An incompressible liquid of density $\rho$ is contained in a vessel of uniform cross-sectional area $A$. If the atmospheric pressure is $p$, then the force acting on a horizontal plane of aren $a$ situated at a depth $d$ in the liquid is given by

A $\quad A p+a d \rho g$
B $\frac{p}{A}+\frac{d \rho g}{a}$
C $\frac{p+d \rho g}{a}$
D $\frac{a}{p+d \rho g}$
E $\quad a(d \rho g+p)$
N78/II/4

28 A steel sphere is held just below the surface of a deep tank of water and released. Which one of the following best illustrates the relationship between the acceleration $a$, and the displacement $z$, of the sphere? [Take $g$ as $10 \mathrm{~ms}^{-2}$.]


29 The force on a glass window in a vacuum system is $2 \times 10^{2} \mathrm{~N}$ when the pressure within the system is 1 Pa and the atmospheric pressure is $1 \times 10^{5} \mathrm{~Pa}$. If the pressure in the system were reduced to $1 \times 10^{-5} \mathrm{~Pa}$, what would be the force on the window?

| A | $2 \times 10^{-3} \mathrm{~N}$ |
| :--- | :--- |
| B | $1 \times 10^{2} \mathrm{~N}$ |
| C | $2 \times 10^{2} \mathrm{~N}$ |
| D | $4 \times 10^{2} \mathrm{~N}$ |
| E | $2 \times 10^{7} \mathrm{~N}$ |

J83/II/3

30 A small metal sphere is released from rest at a height of a few centimetres above the surface of a viscous liquid. On entering the liquid, the sphere experiences a viscous drag proportional to its velocity. Which one of the following graphs most closely represents the variation of acceleration a with time $t$ from the sphere?


31 A sted ball-bearing is released at the surface of a viscous liquid contained in a tall, wide jar. In falling through the liquid, the ball-bearing experiences a retarding force proportional to its velocity. If the depth of the liquid in the jar is $H$, which one of the following graphs best represents the variation of the height $h$ of the ball-bearing above the base with $t$ ?


32 When a beaker of water rests on a balance, the weight indicated is $X$. A solid object of weight $Y$ in air displaces weight $Z$ of water when immersed.


Fig. 8
What will be the balance reading when the object is suspended in the beaker of water so that it is totally immersed as shown in Fig. 8?
A $X$
D $X+Y-Z$
B $X+Z$
E $\quad X+Z-Y$

C $X+Y$
N84/II/8
33 A parachutist steps from an aircraft, falls without air resistance for 2 s and then opens his parachute.

Which graph best represents how a, his vertical acceleration. varies with time $t$ during the first 5 s of his descent?


C



D


J90/1/3; J95/I/3


A sealed U-tube contains nitrogen in one arm and helium at pressure $P$ in the other arm. The gases are separated by mercury of density $\rho$ with dimensions as shown in the diagram. The acceleration of free fall is $g$.

What is the pressure of the nitrogen?
A $P$
B $x \rho g$
C $P-x \rho g$
D $\quad P+x \rho g$
E $\quad P+(x+y) \rho g$
J92/1/23
35 A long narrow tube is filled with water of density 1020 $\mathrm{kg} \mathrm{m}^{-3}$ to a depth of 1.00 m . The tube is then inclined at $30^{\circ}$ to the horizontal as shown.


If atmospheric pressure is 100 kPa , what is the pressure at point X , inside the tube?
A $\quad 5.00 \mathrm{kPa}$
B $\quad 10.0 \mathrm{kPa}$
C $\quad 95.0 \mathrm{kPa}$
D $\quad 105 \mathrm{kPa}$
E $\quad 110 \mathrm{kPa}$
N91/I/21
36 A tall container which is open to the atmosphere contains a layer of liquid $\mathbf{L}$, floating on liquid $\mathbf{M}$. Liquid $\mathbf{M}$ has a density which is twice as great as that of liquid $L$.


Which graph shows how the pressure, $p$, at a point varies with its height, $x$, above the base of the container?


37 An object, immersed in a liquid in a tank, experiences an upthrust.
What is the physical reason for this upthrust?
A The density of the body differs from that of the liquid.
B The density of the liquid increases with depth.
C The pressure in the liquid increases with depth.
D The value of $g$ in the liquid increases with depth.
N2000/I/5
38 A hazard in working with vacuum systems made of glass is implosion, that is, the breakage of the glass and its projection inwards. A certain flat glass window of area $900 \mathrm{~mm}^{2}$ is used as a viewing point in a vacuum system.
(a) What force is exerted by the atmosphere on this window?
(b) If a thickness of 3 mm is judged to be safe against implosion in a vacuum system in which the pressure is 1 Pa , what thickness would be needed if the pressure inside were $1 \times 10^{-5} \mathrm{~Pa}$ ?
[Atmospheric pressure $=1 \times 10^{5} \mathrm{~Pa}$.]
J79/I/2
39 A simple mercury barometer (Fig. 9) indicates a reading of 760 mmHg when used on Earth to measure an atmospheric pressure of $1.0 \times 10^{5} \mathrm{~Pa}$. The pressure of the air in a space laboratory in stable orbit about the Earth is $0.9 \times 10^{5} \mathrm{~Pa}$. Explain why the barometer cannot be used to measure this pressure and state what would happen if an attempt were made to do so.


Fig. 9
J82/I/2

40 A tube is arranged as a siphon in a beaker of liquid (Fig. 10). The point $A$ is 60 mm above the free surface of the liquid and $B$ is 20 mm below it. The tube is first filled with liquid and the end $B$ is closed with a finger.


Find the amounts by which the pressures at $A$ and $B$ are above or below atmospheric pressure.
When the finger is removed from B, liquid runs out. Why is this? When will the flow stop?
[Density of liquid $=0.9 \times 10^{3} \mathrm{~kg} \mathrm{~m}^{-3}$, take the acceleration of frec fall, $g$ as $10 \mathrm{~m} \mathrm{~s}^{-2}$.]

N82/I/3
41 A thin plastic bag is found to have a mass $m$ when empty and pressed flat. When the bag is filled with air at atmospheric pressure and re-weighed, the mass is again found to be $m$. Explain why the two results are the same.

N83/I/2
42 (a) An object has mass of 2.3 kg . What is its weight?
(b) When the object in (ct) falls in air, the air resistance $F$ is given by the equation

$$
F=k v^{2}
$$

where $v$ is the velocity of the object and $k$ has the value $0.042 \mathrm{Ns}^{2} \mathrm{~m}^{-2}$.
(i) Explain why the object eventually falls with a uniform velocity (the terminal velocity).
(ii) Calculate the terminal velocity of the object.
(c) Calculate the acceleration of the object when it is falling with a velocity of $12 \mathrm{~ms}^{-1}$.
[3] J90/II/I
43 (c) One of the divers referred to in (b) had a mass of 61 kg . In a subsequent free-fall dive from an aircraft, before the parachute was opened, the diver reached a terminal speed of $90 \mathrm{~m} \mathrm{~s}^{-1}$. Calculate
(i) the weight of the diver,
(ii) the force of air resistance on the diver when at the terminal speed,

- (iii) the magnitude and unit of the constant $k$ in the expression

$$
\begin{equation*}
\text { air resistance }=k v^{2} \tag{4}
\end{equation*}
$$

where $v$ is the speed.
(d) (i) Use the expression given in (c)(iii) to estimate the air resistance on a diver in (c) moving with a speed of $25 \mathrm{~ms}^{-1}$.

J96/II/1 (part)
44 (a) Define the following.
(i) density
(ii) pressure
(b) A rectangular block of wood of area of cross-section $A$ and thickness $t$ floats horizontally in a liquid as shown in Fig. 11.


Fig. 11

The block floats when its lower face is at a depth $d$ in the liquid of density $\rho$. The block experiences a force $F$ on its lower surface as a result of immersion in the liquid.
(i) State the direction of the force $F$.
(ii) The pressure on the lower surface of the block due to the liquid is $p$. Show that $p$ is related to $d$, $\rho$ and the acceleration of free fall $g$ by the expression

$$
p=d \rho g,
$$

(iii) Using the expression in (ii), show that the force $F$ is related to the volume $V$ of liquid displaced by the expression

$$
\begin{equation*}
F=V r g \tag{5}
\end{equation*}
$$

(c) A fisherman's buoy is held submerged in sea water by a rope anchored to the sea bed as shown in Fig. 12.


The buoy has volume $6.5 \times 10^{-2} \mathrm{~m}^{3}$ and mass 6.0 kg . The mass of the rope may be neglected.
(i) The expression in (b)(iii) can be used to calculate the force $F$, known as the upthrust, on the submerged buoy. The density of sea water is $1.03 \times 10^{3} \mathrm{~kg} \mathrm{~m}^{-3}$. Calculate the value of $F$.
(ii) Show that the tension in the rope is 600 N . [3]

N98/II/I (part)
45 (a) Define density and pressure.
[2]
(b) Use the definitions which you have given in (a) to derive the equation

$$
\begin{equation*}
p=\rho g h \tag{3}
\end{equation*}
$$

for the pressure $p$ at a depth $h$ in a fluid of density $\rho$.
(c) Use your answer to (b) and Fig. 13 to explain why any object submerged in water is easier to lift than when not submerged.


Fig. 13 J2000/II/I

## Long Questions

46 Frictional forces and viscous drag both oppose relative motion. Suggest some similarities and differences between them. J79/I/14 (part)

47 Show that the pressure at depth $h$ below the surface of an incompressible liquid of density $\rho$ is given by $h \rho g$, where $g$ is the acceleration of free fall.
Sketch a graph to show how pressure varies with depth for an incompressible liquid. On the same axes, draw another sketch graph to show how you would expect the pressure to vary with depth if the liquid were compressible. Label your graphs carefully.
The SI unit of pressure is the pascal (Pa): in school laboratories, pressure are olten measured in millimetres of mercury ( mmHg ). Find the conversion factor between mmHg and Pa .

N81/I/15 (part)
48 Explain why the surface of a liquid at rest is plane and horizontal.

Draw a labelled graph to show how pressure $p$ varies with depth $h$ below the surface of an incompressible liquid at rest, taking atmospheric pressure to be $p_{0}$. On the same axes, sketch the pressure variation if the liquid were in a container which was released from rest and allowed to fall freely (i.e. with acceleration $g$ ).

N83/I/16 (part)
49 (b) Derive the equation $\Delta p=\rho g \Delta /$ for the change in pressure $\Delta p$ caused by a change in the depth $\Delta / 2$ in a liquid of density $\rho$.
[2] J91/III/4 (part)
50 In a dynamics experiment, a trolley is accelerated from rest along a horizontal runway as shown in Fig. 14.


Fig. 14
The accelerating force is provided by an elastic cord. One end of the cord is attached to the trolley and the other end is pulled so that the extension of the cord remains constant as the trolley moves along the runway.
The acceleration $a$ of the trolley varies with the extension.$x$ of the elastic cord as shown in Fig. 15.


The trolley experiences a constant frictional force when in motion.
(a) Use Fig. 15 to
(i) determine the extension of the cord required to maintain constant speed of the trolley, giving a brief explanation for your answer,
(ii) show that the increase in extension, beyond that found in (i), to produce an acceleration of $0.60 \mathrm{~m} \mathrm{~s}^{-2}$ is 11.2 cm .
(b) (i) Calculate the force required, in the absence of friction, to cause the trolley of mass 800 g to have an acceleration of $0.60 \mathrm{~m} \mathrm{~s}^{-2}$.
(ii) Using your answers to (b)(i) and (a)(ii), determine the spring constant of the elastic cord. Assume that the cord obeys Hooke's law.
(iii) Calculate the frictional force on the trolley.
(c) In one particular experiment, the extension of the cord is kept constant at 3.5 cm . Calculate
(i) the speed of the trolley after it has travelled 1.2 m from rest along the runway,
(ii) the time taken to travel a further 30 cm along the runway.
(d) By reference to Fig. 15, state and explain
(i) whether the acceleration of the trolley is proportional to the extension of the cord.
(ii) how it may be concluded that the Hooke's law limit of the cord has not been exceeded. [5]

J2000/III/]

## Equilibrium Of Forces

51 A small ball of weight $W$ is suspended by a light thread. When a strong wind blows horizontally, exerting a constant force $F$ on the ball, the thread makes an angle $\theta$ to the vertical as shown.


Which equation correctly relates $\theta, F$ and $W$ ?
A $\cos \theta=F / W$
B $\sin \theta=F / W$
C $\tan \theta=F / W$
D $\tan \theta=W / F$

N83/II/I; J90/I/5; N96/I/6

52 A ladder PQ , resting on a rough floor and leaning against a rough wall, is on the point of slipping. It is of weight $W$ and the contact forces exerted on the ladder by the wall and floor are $X$ and $Y$ respectively. Which one of the following diagrams correctly shows the directions of these forces?


N84/II/I
53 In the diagram below, a body $S$ of weight $W$ hangs vertically by a thread tied at Q to the string PQR .


If the system is in equilibrium, what is the tension in the section $P Q$ ?

A $\quad W \cos 60^{\circ} \cos 30^{\circ}$
B $W \cos 60^{\circ}$
C $W \tan 30^{\circ}$
D $W \cos 30^{\circ}$
E $W \tan 60^{\circ}$
J85/I/2

54 A trailer of weight 30 kN is hitched to a cab the point X as shown in the diagram below.


If the trailer carries a weight of 20 kN at the position shown in the diagram, what upward force is exerted by the cab on the trailer at the point $\mathbf{X}$ ?
A $\quad 15 \mathrm{kN}$
D 40 kN
B $\quad 20 \mathrm{kN}$
E 50 kN

C $\quad 30 \mathrm{kN}$
J86/I/2
55 The diagram below shows a heavy llagpole $P Q$ hinged at a vertical wall at end $\mathbf{P}$ and held by a wire connected between end $\mathbf{Q}$ and a point $\mathbf{R}$ on the wall. The wesght of the flagpole is $W$ and the tension in the wire is $T$


What is the direction of the force exerted by the wall on the flagpole?
A $\overrightarrow{\mathbf{P Q}}$
B
$\overrightarrow{\text { PS }}$
C $\overrightarrow{\mathbf{P X}}$
D $\overrightarrow{\mathrm{QP}}$
E $\underset{\text { SP }}{\overrightarrow{\mathbf{S P}}}$

56 A picture weighing 10.0 N hangs freely by a cord XYZ as shown in the diagram below.


What is the tension $T$ in the cord?
A $\quad 2.9 \mathrm{~N}$
D $\quad 8.7 \mathrm{~N}$
B $\quad 5.0 \mathrm{~N}$
E $\quad 10.0 \mathrm{~N}$
C $\quad 5.8 \mathrm{~N}$

J88/I/1
57 In order to support a load $W$, four light hinged rods $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ and $S$ are connected as shown below and mounted in a vertical plane.


Which rods are in compression and which in tension?

|  | in compression | in tension |
| :--- | :--- | :--- |
| A | $\mathbf{P}$ | $\mathbf{Q}, \mathbf{R}, \mathbf{S}$ |
| B | $\mathbf{P}, \mathbf{Q}$ | $\mathbf{R}, \mathbf{S}$ |
| $\mathbf{C}$ | $\mathbf{Q}, \mathbf{R}$ | $\mathbf{P}, \mathbf{S}$ |
| $\mathbf{D}$ | $\mathbf{Q}, \mathbf{R}, \mathbf{S}$, | $\mathbf{P}$ |
| $\mathbf{E}$ | $\mathbf{R}, \mathbf{S}$ | $\mathbf{P}, \mathbf{Q}$ |



N89/I/I

60 The diagrams show spring balances joined to demonstrate a system of three coplanar forces acting at a point. The readings represent the magnitudes of the forces.
Which system of forces could be in equilibrium?

A


B


C


D


E


J91/1/7

61 A uniform rod XY of weight 10.0 N is freely hinged to a wall at $\mathbf{X}$. It is held horizontal by a force $F$ acting from $\mathbf{Y}$ at an angle of $60^{\circ}$ to the vertical as shown in the diagram.


## What is the value of $F$ ?

A 20.0 N
B 10.0 N
C 8.66 N
D 5.00 N
E 4.33 N N91///6

5 Forces

62 The diagram shows a diving-board held in position by two rods $\mathbf{X}$ and $\mathbf{Y}$.


Which additional forces do these rods exert on the board when a diver of weight 600 N stands on the right-hand end?

|  | $a t \mathbf{X}$ (downwards) | $a t \mathrm{Y}$ (upwards) |
| :--- | :---: | :---: |
| A | 400 N | 1000 N |
| B | 600 N | 1200 N |
| C | 900 N | 600 N |
| D | 900 N | 1500 N |

J92/I/5; J96/I/5
63 A heavy uniform beam of length $l$ is supported by two vertical cords as shown.


What is the ratio $\frac{T_{1}}{T_{2}}$ of the tensions in these cords?
A $\frac{1}{3}$
B $\frac{1}{2}$
C $\frac{2}{1}$
D $\frac{3}{1}$

N92/I/5; J95/I/5
64 A rod of length 1 metre has non-uniform composition, so that the centre of gravity is not at its geometrical centre.

The rod is laid on supports across two top-pan balances as shown in the diagram. The balances (previously set at zero) give readings of 360 g and 240 g .


Where is the centre of gravity of the rod relative to its geometrical centre?
A $\frac{1}{10}$ metre to the left
B $\frac{1}{10}$ metre to the right
C $\frac{1}{6}$ metre to the left
D $\frac{1}{5}$ metre to the right
E $\frac{1}{5}$ metre to the left
J93/I/5

65 A ruler of length 0.30 m is pivoted at its centre. Equal and opposite forces of magnitude 2.0 N are applied to the ends of the ruler, creating a couple as shown.


What is the magnitude of the torque of the couple on the ruler when it is in the position shown?
A 0.23 Nm
C $\quad 0.46 \mathrm{Nm}$
B $\quad 0.39 \mathrm{Nm}$
D 0.60 Nm

N93/I/4; N99/I/5
66 A uniform metre rule of weight 2.0 N is pivoted at the 60 cm mark. A 4.0 N weight is suspended from one end, causing the rule to rotate about the pivot.


At the instant when the rule is horizontal, what is the value of the resultant turning moment about the pivot?

A zero
B $\quad 1.4 \mathrm{~N} \mathrm{~m}$
C $\quad 1.6 \mathrm{Nm}$
D $\quad 1.8 \mathrm{Nm}$
N94/I/5
67 An object is acted on by two forces $P$ and $Q$. A frictional force $F$ holds the object in equilibrium.

Which vector triangle could represent the relationship between these forces?


68 Diagram 1 shows two parallel forces $F$ acting on a bar of length $l$ pivoted at $\mathbf{P}$. The forces give rise to a couple of torque $M$.

In diagram 2, the lines of action of the forces are moved a distance $\frac{l}{4}$ to the left.
diagram 1
diagram 2


What is now the torque of the couple?
A 0
B $\quad M / 2$
C $M$
D $\quad 2 M$
J97/I/5
69 The force diagrams show all the forces acting on a beam of length $3 x$.

Which force system causes only rotational motion of the beam without any linear movement?


A


B


70 The diagram shows the jib of a tower crane. Only three forces act on the jib: the tension $T$ provided by a supporting cable; the weight $W$ of the jib; and a force $P$ (not shown) acting at point $\mathbf{X}$.


The jib is in equilibrium.
Which triangle of forces is correct?


71 The dimensions of torque are the same as those of energy. Explain why it would nevertheless be inappropriate to measure torque in joules. State an appropriate unit.

J85/II/1


Fig. 16

Fig. 17

A picture of weight 5 N is suspended from a hook on a wall by a cord which has a breaking strength of 25 N . Initially (Fig. 16) the picture is found to be too low; the cord is shortened, with the intention of hanging the picture as in Fig. 17. However, when the picture is replaced the cord breaks immediately. Explain why the cord broke when supporting a load so much less then its breaking strength. N85/II/I

73 (a) Coplanar forces $F_{1}, F_{2}, F_{3}$ act on a point mass A , as shown in Fig. 18.


Fig. 18
State, or show on a labelled diagram, the condition for the mass to be in equilibrium.
(b) Coplanar forces, $F_{4}, F_{5}, F_{6}, F_{7}$, which do not all pass through the same point, act on a body $B$, as shown in Fig. 19.


Fig. 19
State, or show on labelled diagrams, the conditions for the body to be in equilibrium.

J87/II//
74 Fig. 20 shows part of the force diagram for the spine of a person bending over, with the back horizontal.


The spine is considered as a rod pivoted at its base. The various muscles of the back are equivalent to a single muscle producing a force $T$ as shown. $W$ is the force that the upper part of the body exerts on the spine.
(a) Explain why, for equilibrium, the value of $T$ is large (typically several times $W$ ).
(b) For equilibrium, a force $P$ at the pivot is necessary. Draw a triangle of forces to show the equilibrium of the spine under the action of forces $T, W$ and $P$. Comment on the size of $P$ relative to $W$.
[6] N88/II/2

75 (a) State the conditions for the equilibrium of a body which is acted upon by a number of forces.
(b) A student holds a uniform metre rule at one end in two different ways, as shown in Figs. 21 and 22 below.


Fig. 21


Fig. 22
On Fig. 21, draw and label an arrow to represent the weight $W$ of the metre rule and an arrow to represent the force $F$ provided by the student's hand. What is the relationship between the magnitudes of $F$ and $W$ ? [3]

In Fig. 22, the rule is held horizontally between the thumb and first finger. On Fig. 22, draw and label all the forces acting on the metre rule. List these forces in order of increasing magnitude.

```
1. .............................
2.
3.
```

J91/II/
76 Fig. 23 shows the driving wheel of a car with a torque of 125 Nm being applied to the axle of the wheel. The car is travelling at a constant velocity of $30 \mathrm{~ms}^{-1}$.


Fig. 23
(a) Show on Fig. 23 the direction in which the car is travelling.
(b) Draw on Fig. 23 an arrow representing $F$, the horizontal component of the force which the road excrts on the tyre.
(c) Since the car is not accelerating, what can be deduced about the resultant torque on the wheel?
(d) Find $F$.
*(e) What power is being supplied to the wheel?

77 (a) Fig. 24 illustrates a force of 180 N acting at a perpendicular distance of 40 cm from a point O .


Fig. 24
Calculate the moment of the force about 0 .
moment $=$ $\qquad$
(b) Fig. 25 illustrates a circular drum of radius 8.0 cm around which a rope is wound. The drum is turned by means of a handle of length 40 cm to which a force of 180 N is applied at right angles to the handle


Fig. 25
Calculate the tension $T$ in the rope.
tension.
$\qquad$ N [2]
(c) A person is pulling a boat on to a trailer using the drum system from (b), known as a winch, as shown in Fig. 26. The person is exerting a force of 180 N on the handle.


Fig. 26
(i) Explain why the force the rope exerts on the boat may be less in practice than the tension you have calculated in (b),
(ii) Calculate the force which the trailer exerts on the axle of the winch when the handle is in the position shown in Fig. 26. State the direction in which this force acts.
$\qquad$
Direction of force $\qquad$ [3]

78 Part of an arch made of stone is shown in Fig. 27.


Fig. 27
The central stone is known as a keystone and has a weight of 600 N . The keystone is supporting a load of 4600 N . The sides of the keystone make an angle of $10^{\circ}$ to the vertical. The two stones P and Q , which are next to the keystone, exert forces at right angles to the sides of the keystone.
(a) On Fig. 27, draw and label arrows to show the following four forces.
$W$, the weight of the keystone
$L$, the force the load exerts on the keystone $P$, the force stone $P$ exerts on the keystone $Q$, the force stone Q exerts on the keystone
(b) In Fig. 28, the vector labelled $L$ represents, to a scale of $1 \mathrm{~cm}: 1000 \mathrm{~N}$, the force the load exerts on the keystone. Complete a vector triangle to the same scale to represent the equilibrium of the four forces listed in (a).

Fig. 28
(c) Use your vector triangle to determine the value of the force $P$.

$$
\begin{equation*}
P=. \tag{2}
\end{equation*}
$$

J98/II/1
79 (d) Currents in the sea water cause the buoy to be displaced so that the rope makes an angle of $35^{\circ}$ with the vertical, as shown in Fig. 29


Fig. 29
The buoy may be considered to be acted upon by three forces, the tension $T$ in the rope, a horizontal force $D$ and a vertical force $V$ equal to 600 N .
(i) Using a scale of 1.0 cm to represent 100 N , complete Fig. 30 to produce a vector triangle for the three forces acting on the buoy.
line of action
of force $D$

> line of action of force $V$

Fig. 30
(ii) Use your vector triangle to determine the magnitude of the force $D$.

$$
\begin{aligned}
& D= \\
& \text { N [4] } \\
& \text { N98/II/1 (part) }
\end{aligned}
$$

80 (a) Explain what is meant by
(i) the moment of a force,
(ii) the torque of a couple.
(b) A desk lamp is illustrated in Fig. 31.


Fig. 31

The lamp must be constructed so that it does not topple over when fully extended as shown in Fig.32. The base of the lamp is circular and has a radius of 10 cm . Other dimensions are shown on the figure. The total weight of the light bulb and shade is 6.0 N and each of the two uniform arms has weight 2.0 N .


Fig. 32
(i) On Fig. 32, draw an arrow to represent the weight of the base.
(ii) The lamp will rotate about a point if the base is not heavy enough. On Fig. 32, mark this point and label it $P$.
(iii) Calculate the following moments about P .

1. moment of first arm

2. moment of second arm
moment $=$.............. $\quad \mathrm{Nm}$
3. moment of light bulb and shade
moment $=$ $\qquad$ Nm
(iv) Use the principle of moments to calculate the minimum weight of base required to prevent toppling.

$$
\begin{array}{r}
\text { weight }=\ldots . . . . . . . . . . . . . . . . N[7]  \tag{7}\\
\mathrm{J} 99 / I I / 3
\end{array}
$$

## Long Questions

81 (d) The cables of a suspension bridge are anchored into large free-standing blocks of concrete as shown in Fig. 33. One of these blocks is shown on a larger scale in Fig. 34; it has a length of 30 m and its cross-section and density are uniform. The maximum force which the cables could exert on this block is $5.5 \times 10^{8} \mathrm{~N}$ for a particular bridge. The force acts in the direction shown so that its line of action is 26 m from the point about which the block might possibly rotate.


Fig. 33

Fig. 34
(i) Sketch Fig. 34 and show the forces which would be acting on the block if it were just about to rotate.
(ii) Calculate the minimum mass of the block needed to prevent rotation when the force exerted by the cable has its maximum value.
(iii) Show on a second sketch the forces which would be acting under normal operating conditions. [10] J89/II/8 (part)

82 (c) A cable car travels along a fixed support cable and is pulled along this cable by a moving draw cable.

(i) For the situation shown, where the cable car can be considered to be stationary and the draw cable exerts negligible force on it, the weight, $W$, of the cable car and passengers is $8.0 \times 10^{4} \mathrm{~N}$. Sketch a vector triangle to show the weight, $W$, of the cable car and passengers and $T_{1}$ and $T_{2}$, the two forces which the support cable exerts on the cable car. Either from a scale diagram, or by calculation, find the magnitude of $T_{1}$.

N92/III/3 (part)
83 (a) What is meant by the centre of gravity of a body?
(b) In a children's game, small balls are thrown at wood blocks in order to turn them over. One such block, of mass 150 g with each side of length 10 cm , is shown in Fig. 35.


Fig. 35


Fig. 36

In order to turn the block over, the centre of gravity C of the block must be raised so that C is vertically above the corner A, as shown in Fig. 36.
(i) For the block as shown in Fig. 36,

1. calculate the vertical height through which the centre of gravity has been raised,
2. show that the gain in potential energy of the block is approximately 0.031 J .
(ii) The block is struck by a ball of mass 11 g travelling horizontally towards C , as shown in Fig 37.


Fig. 37
The collision is perfectly elastic and, without sliding, the block turns about the corner A.

The block is just able to reach the position in Fig. 36. 25\% of the kinetic energy of the ball is transferred to the block. Calculate

1. the kinetic energy of the ball just before it strikes the block,
2. the speed with which the ball strikes the block,
3. the speed with which the ball rebounds from the block.
(iii) For the collision in (ii) of the ball with the block, calculate
4. the change in momentum of the ball,
5. the average force on the block, assuming the ball and the block are in contact for 0.15 s .
(c) A student comments that, by loading the block with an extra mass as shown in Fig. 38, the centre of gravity would be lowered, and consequently, less energy would be required to turn the block.


Fig. 38
Comment on the validity of this statement.

