TOPIC 10 Ideal Gases

1 A large tank contains water at a uniform temperature to a depth of 20 m. The tank is open to the atmosphere and atmospheric pressure is equivalent to that of 10 m of water. An air bubble is released from the bottom of the tank and rises to the surface.

Assuming surface tension effects to be negligible, the volume of the air bubble

A halves before it reaches the surface.
B doubles before it reaches the surface.
C remains constant.
D doubles before it rises 10 m.
E halves before it rises 10 m.

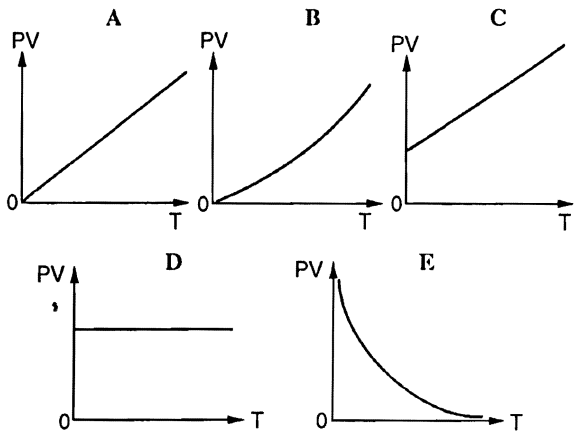
J76/II/26

2 A fixed mass of gas at constant pressure occupies a volume V. The gas undergoes a rise in temperature so that the root mean square velocity of its molecules is doubled. The new volume will be

A $\frac{V}{2}$
B $\frac{V}{\sqrt{2}}$
C $V\sqrt{2}$
D $2V$
E $4V$

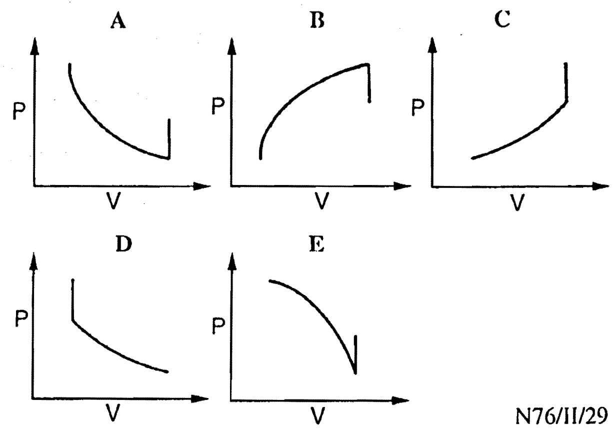
J76/II/36

3 Which one of the following graphs most correctly illustrates the variation of the product (pressure × volume) for an ideal gas with thermodynamic temperature?



N76/II/27; J82/II/26

4 A vessel fitted with a tap contains air at a pressure a few times greater than that of the atmosphere. The tap is opened for a short time and then closed. The vessel is left until the pressure inside becomes constant. Which one of the following graphs represents the variation of pressure with volume for the air left in the vessel?



N76/II/29

5 The r.m.s. speed of hydrogen molecules is v when at a temperature of 300 K. What is its value at a temperature of 450 K?

A $v/1.5$
B $v/\sqrt{1.5}$
C $v\sqrt{1.5}$
D $1.5\,v$
E $2.25\,v$

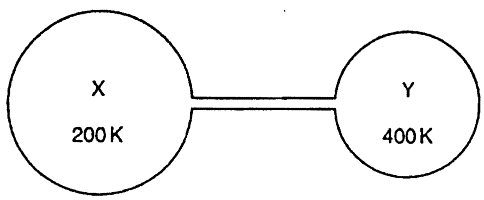
N76/II/37

6 The pressure of a fixed mass of gas at constant volume is greater at a higher temperature because the

A molecules collide with the container walls more frequently.
B number of intermolecular collisions increases.
C molecules travel greater distances between collisions with one another.
D size of each individual molecule increases.
E energy transferred to the walls during collisions increases.

J77/II/39

7 In the diagram the volume of bulb X is twice that of bulb Y. The system is filled with an ideal gas and a steady state is established with the bulbs held at 200 K and 400 K.



There are x moles of gas in X.

How many moles of gas are in Y?

A $\frac{x}{4}$ **B** $\frac{x}{2}$ **C** x **D** $2x$

N77/II/31; J81/II/28; J2000/I/26

8 At a certain temperature, the average speed of the molecules of an ideal gas is c. If the temperature of the gas is changed so that its pressure is halved while keeping its volume constant, then the average speed of the molecules becomes

A $c/4$
B $c/2$
C $c/\sqrt{2}$
D $c\sqrt{2}$
E $2c$ J78/II/38

9 Half a mole of ideal gas occupies x m^3 at a pressure y Pa. R (=8.34 J K^{-1} mol^{-1}) is the molar gas constant. The temperature of the gas is

A $\frac{xy}{8.34}$ K
B $\frac{xy}{16.68}$ K
C $\frac{8.34}{xy}$ K
D $\frac{16.68}{xy}$ K
E $\frac{xy}{4.17}$ K N78/II/27

10 Three particles have speeds of $2u$, $10u$ and $11u$ Which one of the following statements is correct?

A The r.m.s. speed exceeds the mean speed by about $1u$.
B The mean speed exceeds the r.m.s speed by about $1u$.
C The r.m.s. speed equals the mean speed.
D The r.m.s. speed exceeds the mean speed by more than $2u$.
E The mean speed exceeds the r.m.s. speed by more than $2u$.

N78/II/38

11 The temperature at which the r.m.s. speed of nitrogen molecules is twice as great as their r.m.s. speed at 300 K is

A 425 K
B 600 K
C 1146 K
D 1200 K
E 2292 K J79/II/36

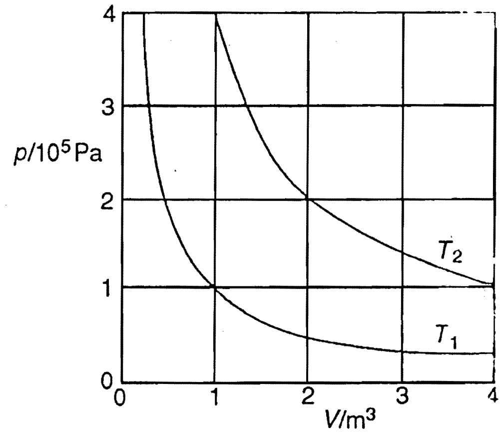
12 The speeds of nine particles are distributed as follows:

speed/m s^{-1}	1.0	2.0	3.0	4.0	5.0	6.0
no. of particles	1	1	4	1	1	1

The root mean square speed is

A 1.2 m s^{-1}
B 3.0 m s^{-1}
C 3.3 m s^{-1}
D 3.6 m s^{-1}
E 10.9 m s^{-1} J79/II/38

13 The two curves shown below are isotherms for a fixed mass of an ideal gas



What is the ratio

$$\frac{\text{r.m.s. speed of the molecules at temperature } T_2}{\text{r.m.s. speed of the molecules at temperature } T_1}?$$

A $\sqrt{2}$ **D** 4
B 2 **E** 16
C $2\sqrt{2}$ J80/II/39; J85/I/23

14 The equation $pV_m = RT$ is the equation of state for

A any mass of ideal gas.
B one mole of ideal gas.
C one mole of a real gas.
D one kilogram of ideal gas.
E one kilogram of monatomic gas. N80/II/30

15 Which one of the following is essential if the equation $pV_m = RT$ is to obeyed well by a real gas?

A Changes should be adiabatic.
B Changes should be isothermal.
C Temperatures should not exceed 273.16 K.
D Volumes should be small.
E Pressures should be low. J81/II/32

16 The root-mean-square speed of the molecules of an ideal gas is c. If the gas is heated at constant volume so that its pressure is increased from p to $3p$, what does the root-mean-square speed become?

A $(^1/_9)\,c$ **D** $3c$
B c **E** $9c$
C $(\sqrt{3})\,c$ J81/II/38; J88/I/25

17 What is the approximate number of atoms in a cubic metre of an ideal monatomic gas at a temperature of 27 °C and a pressure of 1×10^5 Pa?

A 1×10^{22}
B 6×10^{23}
C 2×10^{25}
D 3×10^{26} J82/II/39; J92/I/26; N95/I/24

18 Two vessels X and Y, of volumes V_X and V_Y, are kept at temperatures T_X and T_Y. They are filled with the same ideal gas and are connected by a narrow tube.

What is the ratio $\dfrac{\text{number of molecules in X}}{\text{number of molecules in Y}}$?

A $\dfrac{T_X V_X}{T_Y V_Y}$ **B** $\dfrac{T_X V_Y}{T_X V_X}$ **C** $\dfrac{T_Y V_Y}{T_Y V_X}$ **D** $\dfrac{T_Y V_X}{T_X V_Y}$

N82/II/28; J99/I/25

19 A mixture of two gases at constant temperature contains molecules of two kinds, the first of mass m_1 and r.m.s. speed c_1, the second of mass m_2 and r.m.s. speed c_2. What is the value of c_1/c_2?

A m_1/m_2

B $(m_1/m_2)^{\frac{1}{2}}$

C 1

D $(m_2/m_1)^{\frac{1}{2}}$

E m_2/m_1

N82/II/38

20 An ideal gas exerts a pressure of 60 Pa when its temperature is 400 K and the number of molecules present in unit volume is N. Another sample of the same gas exerts a pressure of 30 Pa when its temperature is 300 K. How many molecules are present in unit volume of this second sample?

A $4N/3$
B $3N/2$
C $3N/4$
D $2N/3$
E $N/2$

J83/II/26; J86/I/23

21 The relationship between pV, the product of pressure p and volume V, and p for a sample of gas at a given temperature is shown by the isotherm marked (i) in the diagram below.

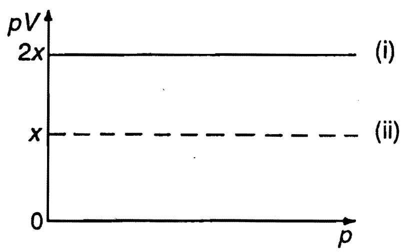


Fig. 2

Isotherm (ii) would apply if

A the mass of gas was doubled.

B the thermodynamic temperature of the gas was doubled.

C the thermodynamic temperature of the gas was halved.

D the mass of gas was doubled and its thermodynamic temperature was halved.

E the mass of gas was halved and its thermodynamic temperature was also halved.

N83/II/28; N86/I/22

22 The average kinetic energy of the molecules of an ideal gas in a closed, rigid container is increased by a factor of 4. What happens to the pressure of the gas?

A It remains the same.

B It increases by a factor of 2.

C It increases by a factor of 4.

D It increases by a factor of 8.

E It increases by a factor of 16.

N83/II/37

23 Argon and neon are monatomic gases with relative atomic masses of 40 and 20 respectively. The ratio

$$\frac{\text{number of atoms of argon in one mole}}{\text{number of atoms of neon in one mole}}$$

is

A always 2.

B 2 only if both gases are at the same temperature and pressure.

C 2 only if the gases are at temperatures where both are solidified.

D always 1.

E 1 only if both gases are at the same temperature and pressure.

N83/II/38

24 The values of pV, the product of pressure and volume, used in the determination of thermodynamic temperature with a gas thermometer are those in which actual measurements have been extrapolated to zero pressure.

This procedure is followed because

A measurements of p and V are more accurate at low pressure.

B extrapolating helps to eliminate errors made in measuring p and V.

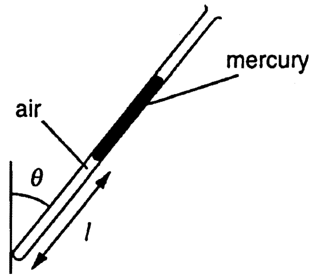
C it was found that temperatures so defined agreed with the established centigrade temperature scale.

D it is impossible to make measurements at 0 K.

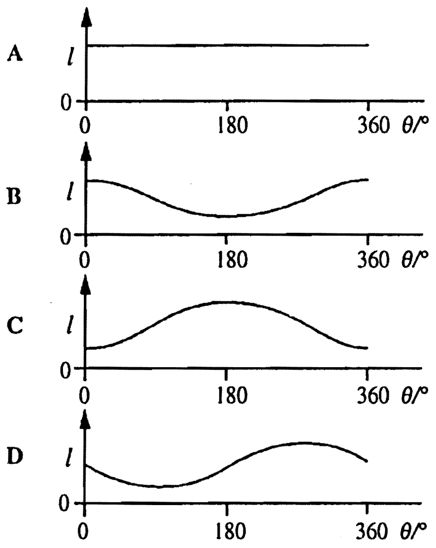
E at near zero pressure at gases behave ideally.

J84/II/26

25 A narrow tube, closed at one end, contains a column of dry air that is trapped by mercury.



Which diagram best shows the variation of the length l of the air column with the angle θ of the tube to the vertical?



J84/II/28; N2000/I/22

26 The temperature of 1 kg of hydrogen gas is the same as that of 1 kg of helium gas if

- **A** the gases have the same internal energy.
- **B** the gases radiate energy at the same rate.
- **C** the gas molecules have the same root mean square speed.
- **D** the gas molecules have the same mean translational kinetic energy.
- **E** the gas molecules occupy equal volumes.

J84/II/38

27 In the expression below, R is the molar gas constant, p is pressure, T is thermodynamic temperature, N_A is the Avogadro constant, n is the amount of substance, k is the Boltzmann constant, and m is the mass of gas. Which one of the expressions is correct for the molar volume V_m of an idea gas?

- **A** RT/p
- **B** N_ART/p
- **C** nRT/p
- **D** nkT/p
- **E** mRT/p

N84/II/26

28 Which one of the following correctly gives the approximate values for the diameter d of a water molecule, its root mean square speed v at room temperature, and the number density n of molecules in water?

	d/m	v/m s^{-1}	n/cm^{-3}
A	3×10^{-12}	20	3×10^{25}
B	3×10^{-12}	600	3×10^{28}
C	3×10^{-10}	600	3×10^{22}
D	3×10^{-8}	20	3×10^{25}
E	3×10^{-8}	600	3×10^{28}

J85/I/22

29 Oxygen molecules in the Earth's atmosphere have a root mean square speed of about 500 m s^{-1}. If the relative molecular mass of oxygen and helium are 32 and 4 respectively, what is the best approximation to the root mean square speed of a helium molecule in the atmosphere?

- **A** 180 m s^{-1}
- **B** 1000 m s^{-1}
- **C** 1400 m s^{-1}
- **D** 2000 m s^{-1}
- **E** 4000 m s^{-1}

N85/I/22

30 In the derivation of the relationship between the pressure, the number density of molecules and the mean square speed of the molecules of an ideal gas, which of the following is *not* an essential assumption?

- **A** that the number of molecules per unit volume is large
- **B** that the molecules are in continuous random motion
- **C** that all collisions are elastic
- **D** that there are no intermolecular forces of attraction
- **E** that the average kinetic energy of a molecule is proportional to the temperature of the gas

J86/I/22; J98/I/25

31 A temperature may be measured by using a constant-volume gas thermometer and measuring the pressure of the gas both at the triple point of water and at the unknown temperature. Which one the following procedures is necessary if two gas thermometers, using different real gases, are to agree?

- **A** Use gases of the same density in both thermometers.
- **B** Use gases of the same specific heat capacity in both thermometers.
- **C** Take readings at very low pressures in both thermometers.
- **D** Use the same mass of gas in both thermometers.
- **E** Use the same volume of gas in both thermometers.

N86/I/23

32 A gas cylinder is fitted with a safety valve which releases a gas when the pressure inside the cylinder reaches 2.0×10^6 Pa. Given that the maximum mass of this gas the cylinder can hold at 10 °C is 15 kg, what would be the maximum mass at 30 °C?

- **A** 5.0 kg
- **B** 14 kg
- **C** 15 kg
- **D** 16 kg
- **E** 45 kg

N87/I/22

33 After the pressure of the air in a bicycle tyre has been increased slightly by pumping air into it, the number of moles of air in the tyre is found to have increased by 2 %, the thermodynamic temperature by 1 % and the internal volume of the tyre by 0.2 %.

By what percentage has the pressure of the air in the tyre increased?

- **A** 0.4 %
- **B** 1.2 %
- **C** 2.8 %
- **D** 3.2 %
- **E** 10 %

J89/I/24; N93/I/24

34 A surface is bombarded by particles, each of mass m, which have velocity v normal to the surface. On average, n particles strike unit area of the surface each second and rebound elastically. What is the pressure on the surface?

A nmv
B $2nmv$
C $\frac{1}{3}nmv^2$
D $\frac{1}{2}nmv^2$
E nmv^2

N89/I/24

35 In a mixture of two monatomic gases X and Y in thermal equilibrium, the molecules of Y have twice the mass of those of X. The mean translational kinetic energy of the molecules of Y is 6.0×10^{-21} J.

What is the mean translational kinetic energy of the molecules of X?

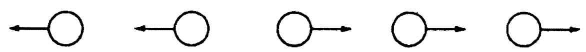
A 3.0×10^{-21} J
B 4.2×10^{-21} J
C 6.0×10^{-21} J
D 8.5×10^{-21} J
E 12×10^{-21} J

N90/I/25

36 In deriving the equation $p = \frac{1}{3}\rho < c^2 >$ in the simple kinetic theory of gases, which of the following is **not** taken as a valid assumption?

A Attractive forces between the molecules are negligible.
B The volume of the molecules is negligible compared with the volume of the gas.
C The duration of a collision is negligible compared with the time between collisions.
D Collisions with the walls of the container and with other molecules cause no change in the average kinetic energy of the molecules.
E The molecules suffer negligible change of momentum on collision with the walls of the container. J91/I/24

37 Five molecules are moving with the speeds and directions shown.



300 m s^{-1} 100 m s^{-1} 100 m s^{-1} 300 m s^{-1} 500 m s^{-1}

What is the root mean square (r.m.s.) speed of these molecules?

A 100 m s^{-1} B 224 m s^{-1} C 300 m s^{-1} D 500 ms^{-1}

N91/I/25; N97/I/25

38 One way of expressing the equation of state for an ideal gas is by the equation

$$pV = NkT$$

What do N and k represent?

	N	k
A	Avogadro constant	Boltzmann constant
B	Avogadro constant	molar gas constant
C	total number of molecules	Boltzmann constant
D	total number of molecules	molar gas constant
E	total number of moles	Boltzmann constart

J93/I/25

39 The simple kinetic theory of gases may be used to derive the expression relating the pressure p to the density ρ of a gas.

$$p = \frac{1}{3}\rho <c^2>$$

In this expression, what does $<c^2>$, represent?

A the average of the squares of the speeds of the gas molecules
B the most probable value of the squares of the speeds of the gas molecules
C the root-mean-square speed of the gas molecules
D the square of the average speed of the gas molecules

J94/I/25

40 Four gas molecules have the speeds shown.

speed / 10^2 ms^{-1}	1.0	3.0	5.0	7.0

What is their root-mean-square speed?

A 2.0×10^2 ms^{-1}
B 2.3×10^2 ms^{-1}
C 4.0×10^2 ms^{-1}
D 4.6×10^2 ms^{-1}

J96/I/25

41 The molecules of an ideal gas at thermodynamic (absolute) temperature T have a root-mean-square speed $c_{r.m.s.}$

The gas is heated to temperature $2T$.

What is the new root-mean-square speed of the molecules?

A $\sqrt{2}\, c_{r.m.s}$
B $2\, c_{r.m.s}$
C $2\sqrt{2}\, c_{r.m.s}$
D $4\, c_{r.m.s.}$

N96/I/25

42 The pressure p of a gas occupying a volume V and containing N molecules of mass m and mean square speed $<c^2>$ is given by

$$p = \frac{1}{3}\frac{Nm}{V}<c^2>$$

The density of argon at a pressure of 1.00×10^5 Pa and at a temperature of 300 K is 1.60 kg m^{-3}.

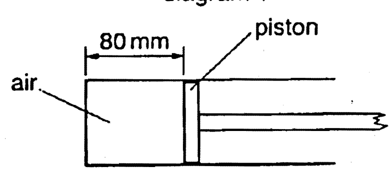
What is the **root** mean square speed of argon molecules at this temperature?

A 216 ms^{-1}
B 250 ms^{-1}
C 306 ms^{-1}
D 433 ms^{-1}

J97/I/25

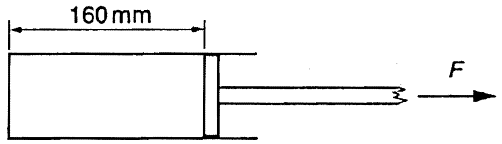
43 Air is enclosed in a cylinder by a gas-tight, frictionless piston of cross-sectional area 3.0×10^{-3} m^2. When atmospheric pressure is 100 kPa, the piston settles 80 mm from the end of the cylinder (see diagram 1).

diagram 1



The piston is then pulled out until it is 160 mm from the end of the cylinder (see diagram 2) and is held there. The temperature of the air in the cylinder returns to its original value.

diagram 2



What is then the force F required to hold the piston in its new position?

A 150 N **C** 300 N
B 200 N **D** 600 N N98/I/25

44 A kinetic theory formula relating the pressure p and the volume V of a gas to the root-mean-square speed of its molecules is

$$p = \frac{1}{3}\frac{Nm}{V} < c^2 >.$$

In this formula, what does the product Nm represent?

A the mass of gas present in volume V
B the number of molecules in unit volume of the gas
C the total number of molecules in one mole of gas
D the total number of molecules present in volume V

N99/I/25

45 A student observes the Brownian motion of smoke particles in air. Which statement explains the movement of the smoke particles?

A Smoke particles are more dense than air molecules.
B There are convection currents in the air.
C The smoke particles are bombarded by moving air molecules.
D The smoke particles collide with one another.

J2000/I/22

46 The simple kinetic theory of gases may be used to derive the expression relating the pressure p to the density ρ of a gas.

$$p = {}^1/_3\, \rho < c^2 >$$

In this expression, what does $< c^2 >$ represent?

A the average of the squares of the speeds of the gas molecules
B the root-mean-square speed of the gas molecules
C the square of the average speed of the gas molecules
D the sum of the squares of the speeds of the gas molecules N2000/I/26

47 One mole of an ideal gas at s.t.p. occupies 2.24×10^{-2} m^3. Assuming that hydrogen behaves as an ideal gas, estimate the average distance between the molecules in a flask of hydrogen at s.t.p.

(The Avogadro constant = 6.02×10^{23} mol^{-1}.) N76/I/10

48 A uniform capillary tube, closed at one end, contained air trapped by a thread of mercury 85 mm long. When the tube was held horizontally, the length of the air column was 50 mm, when it was held vertically with the closed end downwards, the length was 45 mm. Find the atmospheric pressure.

[Take $g = 10$ m s^{-2}; density of mercury = 14×10^3 kg m^{-3}.]

N77/I/8

49 Estimate the number of molecules in a flask of volume 5×10^{-4} m^3 containing oxygen at a pressure of 2×10^5 Pa and a temperature of 300 K.

[Take the molar gas constant R as 8 J mol^{-1} K^{-1} and the Avogadro constant $L(N_A)$ as 6×10^{23} mol^{-1}.]

N78/I/12

50 Taking the molar gas constant to be 8 JK^{-1} mol^{-1}, estimate the molar volume of a gas at 300 K and at a pressure of 1×10^5 Pa.

The volume of a single molecule of the gas is estimated to be 2×10^{-29} m^3. What fraction of the volume occupied by the gas is empty space?

[Take the Avogadro constant, L, to be 6×10^{23} mol^{-1}.]

J80/I/9

51 N molecules of a monatomic ideal gas are contained in a box at pressure p and temperature T. What is the total internal energy of the gas in terms of N, T and the Boltzmann constant k?

An additional N molecules of the same gas are added to the box in such a way as to keep the internal energy constant at its initial value. What happens to *(a)* the temperature, *(b)* the pressure, of the gas? J81/I/11

52 The speed of sound in a gas is given by $c = (\gamma p/\rho)^{\frac{1}{2}}$, where γ is the ratio of the principal heat capacities of the gas, p is its pressure, and ρ is its density. Using the ideal gas equation $pV_m = RT$, derive another expression for c in terms of T and M, the molar mass of the gas. Is it necessary to quote the temperature and pressure in the results of experiments to find the speed of sound? N82/I/5

53 In order to achieve a fusion reaction between deuterium nuclei, temperatures of the order of 1×10^7 K must be attained. Estimate the mean speed of deuterium nuclei at this temperature

[Take the Boltzmann constant, k, as 1.4×10^{-23} JK^{-1} and the mass of a deuterium nucleus as 3.3×10^{-27} kg.]

N82/I/12

54 Find to two significant figures the ratio of the total translational kinetic energy of the molecules in 1.0 mol of an ideal gas at room temperature (17 °C) to the average kinetic energy of a sprinter, a mass 70 kg, while running 100 m in 10 s. N85/II/3

55 *(a)* Find the volume occupied by one mole of an ideal gas at a temperature of 290 K and a pressure of 1.0×10^5 Pa.

(b) The diameter of a molecule of this gas is 2.5×10^{-10} m. What fraction of the volume you have calculated in *(a)* above do the molecules occupy?

J87/II/4

56 An experiment is carried out inside a vacuum system in which the pressure is 1.0×10^{-7} Pa and the temperature is 20 °C. Estimate the number of gas molecules per cubic metre in the system. [3]

N87/II/3

57 A container holds a mixture of hydrogen and oxygen in thermal equilibrium at a temperature of 500 K. Find the mean translational kinetic energies of both types of molecule. Given that the mass of a hydrogen molecule is 3.34×10^{-27} kg and the oxygen molecule is 16 times more massive, find the r.m.s. speeds of both types of molecule. [7]

J88/II/5

58 At the triple point of water the pressure of a fixed mass of gas is 2680 Pa. The temperature is changed to T while the volume of the gas is kept constant. The pressure is then 4870 Pa.

(i) Find the value of T.

(ii) What is the advantage of making this determination at such a low pressure? [5]

J89/II/3

59 An ideal gas has volume 0.50 m^3 at pressure 1.01×10^5 Pa and temperature 17 °C.

(a) (i) Define *pressure.*

(ii) State a unit, other than pascal, for pressure. [2]

(b) Calculate, for this gas, the number of

(i) moles,

number =..

(ii) molecules.

number =.. [5]

(c) Each molecule may be considered to be a sphere of radius 1.2×10^{-10} m. Calculate

(i) the volume of one molecule of the gas,

volume = ...m^3

(ii) the volume of all the molecules.

volume = ...m^3 [2]

(d) (i) State the assumption made in the kinetic theory of gases for the volume of the molecules of an ideal gas.

(ii) Comment on your answer to *(c)*(ii) with reference to this assumption. [3]

J97/II/4

60 *(b)* The gas in a constant volume gas thermometer has a pressure of 3.78×10^4 Pa when the temperature is 273.2 K. Calculate the temperature of the gas when its pressure is 4.03×10^4 Pa, assuming the gas is ideal and its volume is constant. [2]

N97/II/7 (part)

Long Questions

61

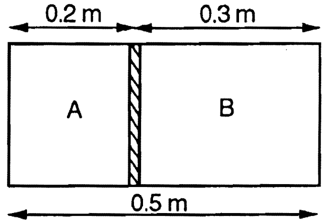


Fig. 1

A cylinder, 0.50 m long and of cross-sectional area S, closed at each end, is fitted with a smooth thermally insulating piston, so as to contain gas in each end A and B (Fig. 1). Initially, the piston is in equilibrium at a distance of 0.20 m from one end of the cylinder, and the gas in each compartment is at a temperature of 27 °C. The temperature of the smaller volume of gas, A, is now increased to 177 °C, while that of the gas in the larger volume B is maintained at 27 °C. The piston moves through a distance x until a new position of equilibrium is obtained.

(a) Write down the equation relating the initial and final pressures, volumes and temperatures of the gas in A.

(b) Write down the equation relating the initial and final pressures and volumes of the gas in B.

(c) Hence find the new position of equilibrium.

N76/I/15 (part)

62 The simple microscopic model of an ideal gas may be used to derive the relation $p = {}^1\!/_3 \rho \overline{c^2}$. By comparing this equation with the ideal gas equation $pV_m = RT$, find an expression for the temperature of a gas in terms of the average kinetic energy of the molecules and the Boltzmann constant.

N76/I/17 (part)

63 *(a)* List the basic assumptions of the kinetic model of a gas.

(b) List the simplifying assumptions usually made to obtain the relation $p = {}^1\!/_3 \rho \overline{c^2}$ between the pressure and the density of an ideal gas.

(c) Explain the meaning of the symbol $\overline{c^2}$. Give the equation relating $\rho \overline{c^2}$ to temperature.

J77/I/16 (part)

64 *(c)* Starting from the ideal gas equation $p = {}^1\!/_3 \rho \overline{c^2}$ and $pV_m = RT$, obtain an expression for the temperature T of an ideal gas in terms of the total translation kinetic energy E of the molecules in one mole.

Estimate the speed at which a person of mass 80 kg would need to run to possess a kinetic energy equal to the total molecular translational kinetic energy of one mole of an ideal gas at room temperature.

[Take the molar gas constant R to be 8 J K^{-1} mol^{-1}.]

J79/I/15 (part)

65 *(a)* Using the ideal gas equation and the kinetic theory equation $p = {}^1/_3\rho\overline{c^2}$, derive an expression for the root-mean-square speed of the molecules of a gas in terms of the molar mass M, the temperature T and the molar gas constant R.

N81/I/17 (part)

66 State the principal assumptions of the kinetic theory of gases.

Without mathematical treatment, use the molecular model of a gas to explain the following phenomena.

(a) The pressure rises if the volume containing a given mass of gas is reduced, the temperature remaining constant.

(b) The temperature rises if the gas is compressed in a thermally-insulated container.

Describe an experiment to investigate the relation between the pressure p and the volume V of a fixed mass of air at room temperature. How would you display your results graphically to demonstrate their agreement (or otherwise) with the relationship that applies for a perfect gas?

The pressure of a mass m of an ideal gas, undergoing the compression described in *(b)* above, increases from p_1 to p_2 as its volume is decreased from V_1 to V_2; the corresponding temperature increase is from T_1 to T_2. Write down equations linking (i) p_1, V_1 and T_1, (ii) p_1, p_2, V_1 and V_2, defining any additional symbols you use.

N82/I/16

67 State two pieces of evidence for believing that matter is made up of molecules.

Write down your estimate of the approximate distance between neighbouring atoms or molecules in a typical solid. Given that gases have densities about a thousand times less than those of solids, what deduction can you make about the approximate distance between molecules in a gas?

The assumptions of the simple kinetic theory of gases include:

(i) gases are made up of many molecules moving at random;

(ii) the collisions of the molecules with the walls of the containers are elastic.

What experimental evidence is there that these assumptions are reasonable?

Use simple kinetic theory to show that the pressure p of an ideal gas is related to its density ρ by the equation $p = {}^1/_3\rho < c^2 >$.

By comparing this equation with the ideal gas equation $pV_m = RT$, find an expression for the temperature T of an ideal gas in terms of the mean-square molecular speed $< c^2 >$ and the molecular mass m. Define any other quantity that appears in your expression.

The temperature of an ideal gas is increased from 27 °C to 28 °C. Find the fractional change in the root-mean-square speed of its molecules.

N84/I/17

68 The N molecules of a gas have speeds v_1, v_2, ... v_N. Write down expressions for (i) the mean speed, (ii) the root-mean-square speed of the molecules.

The mean square speed $< c^2 >$ of the molecules of an ideal monatomic gas is related to the pressure p by the kinetic theory equation $p = {}^1/_3\, nm < c^2 >$.

(a) In this equation, what do the symbols n and m represent?

(b) Starting from this equation, and making use of the ideal gas equation $pV_m = RT$, find an expression for the root-mean-square speed in terms of R, T and the molar mass M of the gas.

(c) Hence find the internal energy U of x moles of the gas at temperature T.

(d) Sketch labelled graphs to show how U depends on (i) the temperature T of the gas, (ii) its volume V, the temperature remaining constant.

A thermally-insulated container is divided into two sections by a thermally-insulating partition. One section contains 3·0 mol of neon at 400 K, and the other 1·0 mol of neon at 600 K. The partition is then removed. Find the equilibrium temperature of the neon, assuming that it behaves as an ideal monatomic gas.

What would be the final equilibrium temperature if, instead of neon, the second section contained 1·0 mol of argon at 600 K? (Assume that argon also behaves as an ideal monatomic gas.) If the root-mean-square speed of the neon molecules is v at this equilibrium temperature, what is the root-mean-square speed of the argon molecules in the mixture?

[Relative atomic masses: neon, 20; argon, 40.]

J85/II/10

69 *(b)* The temperature T of an ideal gas at pressure p is defined by the equation

$$p = nkT.$$

(i) Identify the quantities n and k in this equation.

(ii) Write down another equation from kinetic theory involving p, n, the mean square speed $< c^2 >$ of the molecules, and the mass m of a molecule.

(iii) Hence find an expression for the root mean square speed of the molecules in terms of k, T and m.

A certain plasma contains hydrogen ions (protons) and electrons in thermal equilibrium. Both protons and electrons can be assumed to behave as the molecules of an ideal gas. The root mean square speed of the electrons in the plasma is estimated to be 3×10^6 m s^{-1}.

(iv) What is the root mean square speed of the hydrogen ions?

(v) Estimate the temperature of the plasma.

J86/II/10 (part)

70 One mole of an ideal gas is confined in a cubical container of side *a*. The number density of the molecules (the number per unit volume) is *n*, and the mass of each molecule is *m*. In a simple model the system may be assumed to behave as if at any instant one-sixth of the molecules in the container are moving with speed c directly towards each of the faces of the cube.

(a) Show that the number of molecules hitting a cube face per unit time is $\frac{1}{6} na^2c$.

(b) Hence show that the pressure *p* of the gas is given by

$$p = \tfrac{1}{3}nmc^2.$$

(c) The ideal gas equation is $pV_m = RT$, where V_m is the volume occupied by one mole, *R* is the molar gas constant, and *T* is the absolute temperature. By comparing this equation with that in *(b)* above, show that c is given by

$$c = \sqrt{\frac{3RT}{N_A m}},$$

where N_A is the Avogadro constant.

(d) Use the ideal gas equation to find an expression for the number density *n* in terms of *R, T, p* and N_A.

(e) In a more detailed approach, the mean square speed $< c^2 >$ takes the place of c^2 in the expression for the pressure in *(b)* above. Given two reasons why this is an improvement on the simple theory.

In an experiment to investigate the reaction of oxygen with a clean metal surface, a vessel containing the metal sample was first evacuated to a very low pressure. Oxygen was then allowed to enter the vessel slowly, and some gas molecules became attached to the metal. This process continued until there was an equilibrium between the rate of attachment to the metal and the rate at which the oxygen entered the vessel. When this equilibrium was established, the pressure in the vessel was constant at 1.3×10^{-4} Pa at a temperature of 290 K.

Assuming that the expressions derived above apply to oxygen, find the number of molecules per second hitting the metal surface, of area 40 mm^2.

For every three oxygen molecules incident on the sample surface, one becomes attached to it permanently. If each oxygen molecule thus attached occupies an effective area of 3.0×10^{-20} m^2, estimate the time taken for a uniform layer of oxygen one molecule thick to be formed on the metal.

(Mass of oxygen molecule = 5.3×10^{-26} kg.) N86/II/10

71 *(a)* State the assumptions of the kinetic theory of gases. [5]

(b) The kinetic theory predicts that the pressure exerted by an ideal gas is given by

$$p = \tfrac{1}{3}nm < c^2 >$$

(i) State the physical quantity represented by each term in the equation.

(ii) Use the equation to obtain an expression for the root-mean-square speed of the atoms of a gas in terms of *T*, the temperature of the gas, *M* its molar mass, and *R*, the molar gas constant.

(iii) Calculate the root-mean-square speed of hydrogen molecules at a temperature of −60 °C.

[Molar mass of hydrogen molecules = 2.0×10^{-3} kg mol^{-1}.] [10]

(c) The following passage is a quotation from the *Cambridge Encyclopedia of Earth Sciences.*

In the upper layers of the atmosphere, a large fraction of the hydrogen molecules travel so fast that they are able to escape completely from the Earth. It is estimated that about two-thirds of all the hydrogen molecules present in the atmosphere will escape in about one thousand years. There may also be some loss of helium, but for all other heavier molecules escape is negligible.

Suggest an explanation for each of the following observations.

(i) Fast molecules are able to escape.

(ii) Molecules of small mass are able to escape whereas more massive ones are not.

(iii) Some molecules escape even though the speed required for escape is very much greater than their root-mean-square speed.

(iv) Large quantities of hydrogen have remained indefinitely on the Earth as a constituent of water.

[7] J89/II/10

72 *(c)* Starting from the kinetic theory equation $p = \frac{1}{3}\rho < c^2 >$ and the ideal gas equation $pV = NkT$, deduce an expression, in terms of the thermodynamic temperature *T*, for the average translational kinetic energy of a monatomic molecule. [3]

(d) If two sufficiently energetic sodium atoms collide, an electron in one of them may be raised to a higher energy level.

(i) Explain why such a collision is called an *inelastic collision.*

(ii) Briefly explain why such inelastic collisions are not consistent with basic assumptions of the kinetic theory of gases.

(iii) If the energy required to raise the electron to the higher energy level is 3.35×10^{-19} J, estimate the temperature at which appreciable excitation by collision might take place. [7] J90/III/3 (part)

73 *(a)* 1.00 kg of water contains 3.34×10^{25} molecules.

(i) The density of water at 0 °C and standard atmospheric pressure is 1000 kg m^{-3}. What volume is occupied, on average, by one water molecule? [3]

(ii) The density of steam at 100 °C and standard atmospheric pressure is 0.598 kg m^{-3}. Find the volume occupied, on average, by a molecule in steam at 100 °C and standard atmospheric pressure. [2]

(iii) Estimate, from your answers to (i) and (ii), the ratio of the separation of neighbouring molecules in steam to their separation in water. [2]

(iv) Describe and explain two differences in the microscopic properties of water at 0 °C and steam at 100 °C apart from the change in the separation of the molecules. [4]

J91/III/4 (part)

74 *(a)* State the basic assumptions of the kinetic theory of gases. [4]

(b) The equation $p = \frac{1}{3}\rho \langle c^2 \rangle$ may be derived using kinetic theory.

(i) Explain what is meant by the symbol $\langle c^2 \rangle$.

(ii) Use base units to show that this equation is homogeneous. [4]

(c) The variation with pressure p of the density ρ of a gas was determined at 290 K and at another temperature T. The results are shown in Fig. 2.

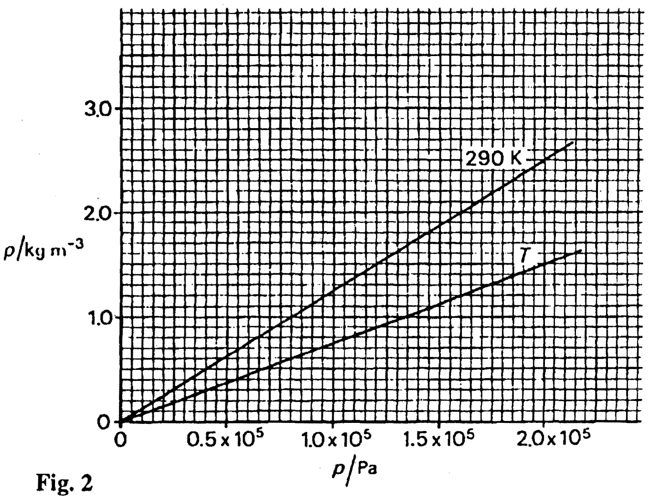


Fig. 2

(i) By reference to Fig. 2, comment on whether the gas behaves as an ideal gas. [3]

(ii) Determine the root-mean-square speed of the molecules of the gas at 290 K. [3]

(iii) Deduce whether the temperature T is greater or less than 290 K. [3]

(iv) Calculate the temperature T. [3]

N93/III/4

75 *(d)* (ii) The pressure p of an ideal gas of density ρ is related to the mean square speed $\langle c^2 \rangle$ of its molecules by the expression

$$p = \frac{1}{3}\rho \langle c^2 \rangle$$

Deduce an expression for the thermodynamic temperature T of the gas in terms of the mean kinetic energy $\langle E_k \rangle$ of a molecule at that temperature. [5]

J94/III/6 (part)

76 *(a)* Some cars are fitted with bags packed into the steering column. In an accident, gas is forced under pressure into the bag and the bag of gas quickly acts as a cushion between the driver and the steering wheel. In one such system, the volume of gas used in the bag is 0.037 m^3 when the pressure is 1.8×10^5 Pa and the temperature of the gas is 6 °C. Calculate

(i) the temperature of the gas in kelvins,

(ii) the amount of gas used, in moles,

(iii) the pressure in the bag when the temperature rises to 18 °C assuming the volume to remain constant while the temperature rises. [5]

(b) Explain why the use of the bag described in *(a)* can reduce injuries. [2]

(c) Consider a cubical box of side l which contains N molecules, each of mass m, all moving horizontally with speed u at right angles to wall A. See Fig. 3.

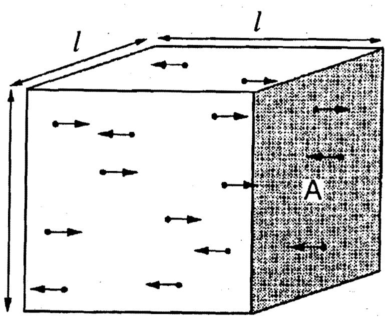


Fig. 3

When a molecule hits a wall, it bounces off with no loss of speed and travels in the opposite direction. Deduce

(i) the momentum of a molecule just before a collision with the wall,

(ii) the change in momentum of a molecule when it collides with the wall,

(iii) the time taken by one molecule between collisions with wall A,

(iv) the total number of collisions per unit time made with wall A by all the molecules,

(v) the rate of change of momentum for all the molecules colliding with wall A. [7]

(d) Use your answer to *(c)* to show that the pressure p on wall A is given by

$$p = \frac{Mu^2}{V}$$

where M is the total mass of all the molecules and V is the internal volume of the box. [2]

(e) The conditions considered in *(c)* are highly improbable. Explain briefly how the conditions may be altered to provide a better model of an ideal gas. State, without proof, how the equation in *(d)* might be modified. [4]

N94/III/5

77 *(a)* State the basic assumptions of the kinetic theory of gases. [4]

(b) An ideal gas is allowed to expand suddenly, with no thermal energy entering or leaving the gas. The temperature of the gas is observed to change.

*(i) Explain, by reference to the first law of thermodynamics, what happens to the internal energy of the gas.

(ii) Explain why the change in the internal energy of the gas will give rise to a change in the r.m.s. speed of the molecules of the gas.

(iii) Hence explain whether the temperature of the gas will rise or fall as a result of this expansion. [8]

N95/III/5 (part)

78 *(b)* (i) What is meant by an *ideal* gas?

(ii) The pressure p of an ideal gas of density ρ is given by the expression

$$p = \tfrac{1}{3}\rho < c^2 > .$$

1. Identify the quantity $< c^2 >$.

2. Deduce an expression for the average translational kinetic energy of a gas molecule in terms of the thermodynamic temperature T. [5]

(c) The air cylinder for a diver has a volume of 9.00×10^3 cm^3 and when the cylinder is filled, the air has a pressure of 2.10×10^7 Pa at 24 °C. The diver is swimming in water of density 1.03×10^3 kg m^{-3} and temperature 24 °C at a depth of 15.0 m. When the diver breathes in, the pressure of the air delivered from the cylinder to the diver is always equal to the pressure of the surrounding water.

Atmospheric pressure is 1.01×10^5 Pa.

Calculate, for the depth of 15.0 m,

(i) the total pressure on the diver,

(ii) the volume of air available at this pressure from the cylinder. [4]

(d) The supply of air in *(c)* is sufficient for the diver to remain at a depth of 15.0 m for 45 minutes. Assuming that the diver always breathes at the same rate (i.e. the same volume of air is required per minute, regardless of pressure), how long would the air in the cylinder last for the diver at a depth of 35.0 m and a water temperature of 20 °C? [4]

J96/III/5 (part)

79 *(b)* (i) The pressure p in an ideal gas is given by the expression

$$p = \frac{1}{3}\frac{Nm}{V} <c^2>.$$

State the meaning of each of the symbols in the equation.

(ii) Using the equation in (i) and $pV = nRT$, deduce that the internal energy of an ideal gas is given by

internal energy $= \frac{3}{2} NkT.$ [7]

J98/III/5 (part)

80 *(a)* Define the term *density*. [1]

(b) Outline how molecular movement causes the pressure exerted by a gas. [2]

(c) One mole of oxygen has a mass of 32 g. Assuming oxygen behaves as an ideal gas, calculate

(i) the volume occupied by one mole of oxygen gas when at temperature 273 K and pressure 1.01×10^5 Pa,

(ii) the density of oxygen gas at this temperature and pressure. [5]

(d) (i) Explain what is meant by the root-mean-square speed $\sqrt{<c^2>}$ of gas molecules.

(ii) Calculate the root-mean-square speed of four molecules travelling with speeds 300 m s^{-1}, 400 m s^{-1}, 500 m s^{-1} and 600 m s^{-1}. [4]

(e) Assuming ideal gas behaviour, calculate for oxygen at 273 K

(i) the root-mean-square speed of its molecules,

(ii) the average kinetic energy of a molecule. [4]

(f) Oxygen has a boiling point of 90 K and a melting point of 55 K. Describe qualitatively how oxygen at 273 K and oxygen at 27 K differ in respect of

(i) density,

(ii) spacing of the molecules,

(iii) order in the pattern of molecules,

(iv) motion of the molecules. [4]

N99/III/4

81 *(b)* The pressure p of an ideal gas of density ρ is related to the mean-square speed $<c^2>$ of its molecules by the expression

$$p = \frac{1}{3}\rho < c^2 >.$$

(i) State three basic assumptions of the kinetic theory of gases, which lead to a model of an *ideal* gas. [3]

(ii) Write down the equation of state for an ideal gas. [1]

(iii) Show that the average kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature T. [4]

(c) Free neutrons in the core of a fission reactor are sometimes referred to as a 'neutron gas'. These free (thermal) neutrons may be assumed to behave as molecules of an ideal gas at a temperature of 35 °C.

(i) Calculate, for a free neutron of mass 1.67×10^{-27} kg,

1. its mean kinetic energy,

2. its root-mean-square (r.m.s.) speed.

(ii) Determine the temperature of helium gas, assumed to be an ideal gas, at which helium molecules (each of mass $4u$) would have the same r.m.s. speed as the free neutrons. [6]

J2000/III/5 (part)

82 *(d)* The equation $T = m\langle c^2\rangle / 3k$ gives the temperature T of molecules in terms of their mass m, their mean square speed $\langle c^2\rangle$ and the Boltzmann constant k. Using this equation for the oxygen and hydrogen molecules before the collision in *(b)*, gives a value of T for the hydrogen molecule of 283 K, and for the oxygen molecule of 210 K.

(i) Explain why it is difficult to justify the application of this equation to this situation.

(ii) How is it possible for the two molecules in the gas to have different values of T before the collision? [4]

N2000/III/1 (part)

83 *(a)* Derive from the definitions of density and pressure, the equation $p = \rho gh$ for the pressure exerted by a column of liquid of height h and density ρ. [3]

(b) (i) Using the kinetic model of a gas, explain how a pressure is exerted by a gas.

(ii) Calculate the root-mean-square speed of gas molecules in a gas at pressure 1.05×10^5 Pa and of density 1.29 kg m^{-3}. [5]

(c) (iii) An ideal gas is defined as one for which, at constant pressure, the volume of the gas is proportional to the absolute temperature. Calculate the absolute temperature T when an ideal gas has volume 0.00783 m^3, assuming that the same mass of the ideal gas had volume 0.00308 m^3 when at the same pressure and at temperature 273 K. [2]

N2000/III/5 (part)